RIGOUR IN THE MATHS CLASSROOM

Highlights from *Rigor in the K5 Math Classroom",*
Barbara Blackburn and Abigail Armstrong, 2020

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*Rigour is creating an environment in which each student is expected to learn at high levels; each student is supported so he or she can learn at high levels; and each student demonstrates learning at high levels (*Blackburn, 2012, "Rigor is Not a Four-Letter Word"

This trifold approach to rigour described by Barbara Blackburn is not limited to the curriculum that students are expected to learn. It is more than a specific lesson or instructional strategy. It is deeper than what a student says or does in response to a lesson. Rigour includes the environment you create and is the result of weaving together all elements of schooling to raise students to higher levels of learning (p8)

Why rigour in maths?

In 2015 the US the National Council for Teaching Mathematics (NCTM) found that one of the issues that hinders student progress in Mathematics is a lack of rigour.

It found that two issues facing mathematics classrooms are that students are learning isolated facts and procedures with no conceptual understanding, and the lower expectations for students who are considered on a lower academic level. It recommended three shifts for mathematics classrooms to ensure Focus, Coherence and Rigour (see table opposite) (p10)

Mathematics	
Delving Deeply Into the Key Processes and Ideas Upon Which Mathematical Thinking Relies	
Shift	*Explanation*
Focus: Focusing strongly where the standards focus	Focusing deeply on the major work of each level will allow students to secure the mathematical foundations, conceptual understanding, procedural skill and fluency and ability to apply the math they have learned to solve all kinds of problems—inside and outside the math classroom.
Coherence: Designing learning around coherent progressions level to level	Create coherent progressions in the content within and across levels so that students can build new understanding on previous foundations. That way, instructors can count on students having conceptual understanding of core content.
Rigor: Pursuing conceptual understanding, procedural skill and fluency and application—all with equal intensity	Conceptual understanding of key concepts, procedural skill and fluency and rigorous application of mathematics in real-world contexts.

Raise Expectations – Use higher-level questioning

Almost every teacher or leader I talk with says, "We have high expectations for our students." Sometimes this is evidenced by the behaviours in the school; other times, however, actions don't always match the words. As you design lessons that incorporate more rigorous opportunities for learning, you will want to consider the questions that are embedded in the instruction. Higher-level questioning is an integral part of a rigorous classroom.

***It is also important to pay attention to student answers**. When we visit schools it is not uncommon to see teachers who ask higher level questions. But, for whatever reason we then see that some of the teachers accept low level responses from students. In rigorous classrooms, teachers push students to respond at high levels. They ask extending questions, ones that encourage students to explain their reasoning and think through ideas. When a student does not know the immediate answer but has sufficient background information to provide response, the teacher continues to probe and guide the student's thinking rather than moving on to the next student.*

Teachers with high expectations insist on students thinking and problem solving. They also support those students who need it with additional scaffolding. This requires that teachers ask themselves during every step of their lessons, "What extra support might my students need?" (p10)

There are several strategies you should incorporate as you question students:

1. *Provide adequate wait time*
2. *Call on a variety of students, not just those who raise their hands*
3. *Ask higher-order questions*
4. *If you ask a lower-order questions, follow-up with a higher-order question*
5. *Encourage follow-up questions from students*
6. *If a student struggles with an answer, provide guidance and scaffolding rather than moving to another student.*

Use a Round Robin Process

Another time to focus on the process of questioning is during classroom discussions. Rather than asking a question, stating the answer is correct, and then moving on – use a round-about process.

1. *For the first round, simply take all possible responses.*
2. *For the second round, ask students to partner with another student and discuss the responses from the class. They should agree on the best possible response.*
3. *In the third round, discuss the partners' picks for answers and agree upon an answer.*
4. *Finally, reflect on the process, with a focus on what helped students decide on the best answer.*

This process takes a bit of time, so you don't need t do it every time you ask a question; instead use it periodically when a "big idea" is the focus of the learning (pp17 & 18).

Objective-Based Questioning

Faculty at Washington University organise open-ended questions around twelve objectives. Examples are shown in the table opposite. It's important that you think about the readiness level of your students and make adjustments as required (p99).

Objective-Based Questions for Math

Objective	*Description*
To prompt students to investigate a thought process	♦ What are the assumptions that informed the design of this experiment? ♦ What assumptions is your reasoning built on in the mathematical proof?
To ask students to predict possible outcomes	♦ What might happen if you used XXX strategy to solve the problem? ♦ Would you get a different result?
To prompt students to connect and organize information	♦ How does this article explain more about what we studied last week? ♦ Can you develop a graph or table that organizes this information in a helpful way?
To ask students to apply a principle or formula	♦ How does this principle apply to the science investigation you completed? ♦ Who can suggest how we might use this new formula to solve the problems we examined at the start of class today? ♦ When is your solution not valid or does not make sense?
To ask students to illustrate a concept with an example	♦ Can you think of another real-life example of what happened during your experiment? How does your research support your example? ♦ Can you point us to a specific part of the problem that led you to that conclusion? ♦ Can you identify a table, graph or design that exemplifies that idea?
To prompt students to support their assertions and interpretations	♦ How do you know that? ♦ Which part of the text led you to that conclusion?

DEMONSTRATIONS OF LEARNING

The third component of a rigorous classroom is providing each student with opportunities to demonstrate learning at high levels. There are two aspects of students' demonstration of learning. First, we need to provide rigorous tasks and assignments for students, but then what we've learnt is that we also need to provide opportunities for students to demonstrate that they have truly mastered more than a basic lesson.

We prefer the framework of Webb's Depth of Knowledge to set the criteria. Contrasting with traditional methods of evaluating the content of student assessments, the DOK framework is designed to measure content complexity, not difficulty. This process is the most effective way to evaluate content to align with learning expectations and assessments as standards continue to evolve and set new expectancies.

Where do the tasks you set fall on this framework? For more information on how this framework can be used read the https://www.edutopia.org/article/how-use-norman-webb-depth-of-knowledge

SUBJECT	SUMMARY DEFINITIONS OF DEPTH OF KNOWLEDGE (DOK)			
	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
Mathematics	Requires students to recall or observe facts, definitions, and terms. Includes simple one-step procedures. Includes computing simple algorithms (e.g., sum, quotient). *Examples:* • Recall or recognize a fact, term, or property. • Represent in words, pictures, or symbols a math object or relationship • Perform a routine procedure, such as measuring • At higher grades, solve a quadratic equation or a system of two linear equations with two unknowns	Requires students to make decisions on how to approach a problem. Requires students to compare, classify, organize, estimate, or order data. Often involves procedures with two or more steps. *Examples:* • Specify and explain relationships between facts, terms, properties, or operations • Select procedure according to criteria and perform it • Use concepts to solve routine multiple-step problems.	Requires reasoning, planning, or use of evidence to solve a problem or algorithm. May involve an activity with more than one possible answer. Requires conjecture or restructuring of problems. Involves drawing conclusions from observations, citing evidence and developing logical arguments for concepts. Uses concepts to solve non-routine problems. *Examples:* • Formulate original problem, given situation • Formulate mathematical model for complex situation • Produce a sound and valid mathematical argument • Devise an original proof • Critique a mathematical argument	Requires complexity at least at the level of DOK 3 but also an extended time to complete the task. A project that requires extended time but repetitive or lower-DOK tasks is not at Level 4. Requires complex reasoning, planning, developing, and thinking. May require students to make several connections and apply one approach among many to solve the problem. May involve complex restructuring of data, establishing and evaluating criteria to solve problems. *Examples:* • Apply a mathematical model to illuminate a problem, situation • Conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results • Design a mathematical model to inform and solve a practical or abstract situation

Revised 2014

THINK LIKE A MATHEMATICIAN

It is important that our students understand how mathematicians think. Josh Bernoff in "Groundswell" finds that if you are thinking like a mathematician you:

- *Recognise that all reasoning depends on assumptions,*
- *Believe you could be wrong,*
- *Value intuition and ideas,*
- *Question numbers, and*
- *Model things*

Thinking from the perspective of a mathematician or a scientist focuses students' attention on the nuances of the content area. For example, if a task asks students to analyse a natural disaster, a mathematician will look at the data chronicling the natural disaster, while a scientist would consider the environmental impact and implications (p117)

Carol Lloyd also developed a model for guiding mathematical thinking

GUIDED THINKING	
Guided Question	**Explanation**
How do you know what you know?	Students use resources to justify their thinking eg "I need to know or use my understanding of ….. in order to solve this problem."
What is influencing your thinking?	Students analyze their point of view so that they understand there may be various ways to solve a problem. "To solve this problem I …..”
So what do you understand? And why is this concept or practice important?	"So what" statements should help students do two things: understand the content standard and the learning process they used. The "so why" statements allow students to make real life connections.

EMBED REASONING

The mathematical process of reasoning is at the heart of thinking like a mathematician. Reasoning is an essential component of rigorous mathematical learning.

Students from a young age can begin to reason. If you want a template for students to use as they move through the process, one of the simplest ways for students to practice writing proofs is to use a two column proof. The left hand column includes statements used to prove the information, and the right-hand side is the reason or justification.

Examining reasoning by writing mathematical proofs is one way to increase rigour in mathematics. As students provide logical arguments for mathematical solutions and examine their own answers for reasonableness, they exercise their critical-thinking skills.

Name ______________________ Date ______________________
Math Test ___________________ Teacher ____________________

Question Missed	*My Original Answer:*	*My New Solution (you must show your work including all steps):*	*The Correct Answer:*	*Why I Know I Have the Right Answer Now:*

Why I Missed the Question on the Original Test (circle all that applies):

- I didn't understand the question.
- I thought I had it right.
- I skipped a step.
- I studied this, but I forgot.
- I had no clue about this.
- I ran out of time or guessed.
- I made a careless mistake.

Two Column Proofs	
Statement	*Reason*
(a+b)+c = a +(b+c)	Given from the board
(5 +6) +3 = 5 +(6 +3)	Parentheses have been added first because of order of operations, 11 +3 =14 and 5+9 = 14
5 + (6 +3) = (5 + 6) =3	I flipped the problem and still got 14 on both sides
(a+b)=c = a +(b +c)	No matter how I add the numbers, the sum is the same. This is the associative property

Owning and Overcoming Mistakes

One specific opportunity you can provide to students to give them more ownership is to allow them to revisit their formal assessments or practice tests. When we mark students' work we sometimes penalize them for their errors and had the paper back to be filed or thrown away. Because of this, students do not typically learn from their mistakes.

A better way is to use a reflective tool after students have taken the test. In the sample opposite, students are provided an opportunity to redo their mistakes, but they are also required to identify what caused the mistake.

Students need time to revisit errors in order to overcome them and prevent them from recurring repetitively. This is the essence of rigorous expectations. (pp28,29)

Developing Student Ownership

Students deeply desire having freedom of choice. You can integrate "pause points" into your sequence of maths lessons at which students get to chose how, when, or what they learn.

The options for choice may revolve around adjusting the content, support and scaffolding, your instruction, performance of understanding, ***as long as the learning objective remains intact and each task is equally rigorous*** *(pp30 & 31).*

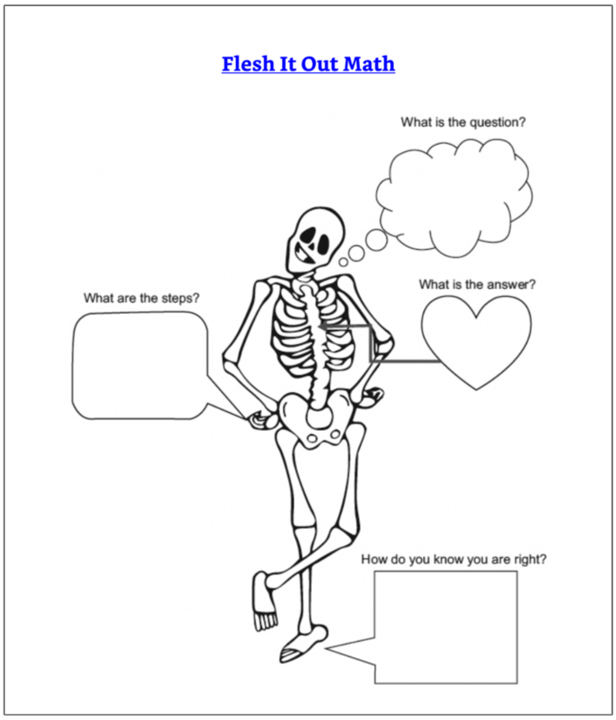
Choice Options: Math		
Solving Problems: Math		
Support and Scaffolding	*Demonstration of Learning*	*Expectations*
Students may choose to . . . ♦ watch a short video on the concept ♦ ask another student to explain ♦ work alone, with a partner or in a small group ♦ do a center activity to provide content support	Students may choose . . . ♦ which strategy they use to solve the problem ♦ how to present final information using the format of their choice (oral or written response)	Students may choose . . . ♦ the resources for foundational knowledge. ♦ how they will approach the problem ♦ the center they want to visit for enrichment

How could you move to deeper levels of thinking in your lessons?

Flesh it Out

A fun visual to help students go deeper with a topic, "Flesh It Out", was originally developed by Janet Allen, and requires an in-depth analysis of a problem or investigation.

Rather than simply writing basic information, students are expected to describe more specific information, which allows them to create a finished problem with more complexity. In this example, students write the question given and their answer; they detail the steps they used; and, to up the level of thinking, write their reasoning (How do you know you were right?)



Math Examples		
Type of Transfer	*Low Road*	*High Road*
Academic	Compute the answer for 359 ÷ 6	How can you use your knowledge of long division to figure out how much money each of your classmates will receive to spend at the souvenir shop if your class raised a certain amount of money?
Real World	What is a real-life example of a problem in which you would use long division?	Your grandmother won $1000, and she decided to split it with her 8 grandchildren. How much money would each of you, each grandchild, receive? Show various ways you can figure out the answer.
Science Examples		
Type of Transfer	*Low Road*	*High Road*
Academic	How did the information on the water cycle relate to what you know about cloud formation?	Next year, we will be studying changes in the Earth's land. How do you think our unit on landforms will prepare us for that content?
Real World	How does the water cycle relate to us when it rains?	How does a drought or a flood affect the water cycle?

Transfer Learning

John Hattie talks about learning at three levels: surface learning, deep learning and transfer learning.

In their book, Tools for Teaching Conceptual Understanding, Julie Stern & Krista Ferraro, discuss transfer learning. They distinguish the transfer as academic or real world and high road- transfer to highly dissimilar tasks or scenarios (more rigorous) or low road – transfer to highly similar tasks or real world scenarios (less rigorous).

This is illustrated in the table opposite for both Maths and Science.

How could you create learning tasks using the High Road for the units your class is studying?

Incorporating inquiry

Through the inquiry process, students are prompted to think through and explain the process rather than simply supplying an answer. These processes might also require transfer and application of learning to problems with real-life connections.

The following website provide resources:

1. Gallileo.org. for quality inquiry-based lesson plan ideas
   https://galileo.org/classroom-examples/
2. PBL Works. Twenty extended project ideas for maths that cover a range of grade level
   https://my.pblworks.org/projects?f%5B0%5D=subject_projects%3A77

PROVIDE REGULAR TIME FOR STUDENT REFLECTION

Reflection should be a routine component part of the rigorous classroom. Students may need starter prompts to guide the reflective process.

Sample Reflection Prompts (Sample Self-Reflection in Parentheses)

- Why do you think ...? (I wonder why ...?)
- How did you decide ... ? (I did this because ... ?)
- Have you considered ... ? (What if I ... ?)
- What would you suggest for ... ?
- How might a mathematician ... ?
- What was your intention when ... ?
- What is the connection between _____ and _____?
- What criteria did you use to ... ? (What criteria did I use to ... ?)

REFLECTING ON YOUR OWN EXPECTATIONS

This final activity requires an inquiry mindset, but should provide you with real insights into how you could genuinely develop higher expectations in your teaching.

Blackburn & Armstrong cite the work of Williams (2012) in arguing that teachers' beliefs, reflected in actions, demonstrate their expectations for their students. In other words, teachers treat students differently dependent on what they expect. Although the difference in treatment may not be intentional, students notice it and will meet those expectations – no matter how high or low they are.

Robert Marzano (2010) identified typical behaviours related to low & high expectations of students as shown in the table below.

MARZANO TYPICAL BEHAVIOURS	Affective Tone	Academic Content Interactions
NEGATIVE	• Less eye contact • Smile less • Less physical contact • More distance from student's seat • Engage in less playful or light dialogue • Use comfort talk ("That's okay, You can be good at other things") • Display angry disposition	• Call on less often • Provide less wait time • Ask less challenging questions • Ask less specific questions • Delve into answers less deeply • Reward them for less rigorous responses • Provide answers for students • Use simpler modes of presentation and evaluation • Do not insist that homework is completed • Use comments eg, "Wow, I'm surprised that you answered that correctly."
POSITIVE	More eye contact Smile more	Call on more often Provide more wait time

	More physical contact Less distance from student’s seat Engage in more playful or light dialogue Little use of comfort talk	Delve into answers more deeply Reward them for more rigorous responses Use more complex modes of presentation and evaluation Insist that homework is completed Use more praise

Marzano also provided a four-step process to identify expectation behaviours and to redress them.

STEP 1: Identify 3 or 4 students for whom you have low expectations. This is not an easy task because it requires that you to admit that you have formed negative expectations about students

STEP 2: Identify similarities in students. This is the most difficult part of the strategy because none of us likes to admit thar we automatically form conclusions about certain types of students. For example, a teacher may find that the students for whom she has low expectations all tend to look a certain way or peak a certain way.

STEP 3: Identify differential treatment of low-expectancy students. In practice, teachers’ behaviours toward students are much more important than their expectations. Use the lists above to guide your reflection. What different behaviours are you using for your low expectations students as opposed to other students?

STEP 4: Treat low-expectancy and high-expectancy students the same. Make a conscious effort for one week to change your behaviour in relation to your targeted low expectations students.

Marzano cautions to be sure that you are changing academic content interactions as well as affective tone. It is fairly easy to establish a positive affective tone with all students.

Providing equal treatment is more difficult when it comes to academic interactions, however, particularly when questioning students. Students for whom teachers have low expectations become accustomed to the teacher asking them fewer and less challenging questions than other students. When teachers change this behaviour, some students might feel uncomfortable. They will probably need to go through the uncomfortable phase to arrive at a place where they put forth new ideas.

Addressing the issue of low expectations and differential treatment is a powerful strategy to enhance the achievement of those who do not traditionally do well. One of the more challenging aspects of effective teaching is confronting one’s own expectations openly and productively.