CHAPTER 5

Rich Mathematical Tasks

Teachers are the most important resource for students. They are the ones who can create exciting mathematics environments, give students the positive messages they need, and take any math task and make it one that piques students' curiosity and interest. Studies have shown that the teacher has a greater impact on student learning than any other variable (Darling-Hammond, 2000). But there is another critical part of the mathematics learning experience—in many ways, it is a teacher's best friend—and that is the curriculum teachers get to work with, the tasks and questions through which students learn mathematics. All teachers know that great mathematics tasks are a wonderful resource. They can make the difference between happy, inspired students and disengaged, unmotivated students. The tasks and questions used help develop mathematical mindsets and create the conditions for deep, connected understanding. This chapter will delve into the nature of true mathematics engagement and consider how it is brought about through the design of mathematics tasks.

I have taught mathematics at middle school, high school, and undergraduate levels in England and the United States. I have also observed and researched hundreds of mathematics classrooms, across the K–16 level, in both countries, and studied students' learning of mathematics and the conditions that bring it about. I am fortunate to have had such a broad experience for many reasons, one of them being it has given me a great deal of insight into the nature of true mathematics engagement and deep learning. I have witnessed mathematical excitement, as it happens, with a range of different students, leading to the development of precious insights into mathematical ideas and relationships. Interestingly, I found that mathematics excitement looks exactly the same for struggling 11-year-olds as it does for high-flying students in top universities—it combines *curiosity*, *connection making*, *challenge*, and *creativity*, and usually involves *collaboration*. These, for me, are the 5 C's of mathematics engagement. In this chapter I will share what I have learned about the nature of mathematics engagement and excitement before considering the qualities of tasks that produce such engagement—tasks that all teachers can create in their own mathematics classrooms.

Rather than dissecting the nature of mathematics engagement in a clinical and abstract way, I want to introduce you to five cases of true mathematics excitement. I think of mathematics excitement as the pinnacle of mathematics engagement. These are all cases that I have personally witnessed among groups of people and that have given me important insights into the nature of the teaching and tasks that bring about such learning opportunities. The first case comes not from a school but from the unusual setting of a startup company in Silicon Valley. This case shows something powerful about mathematical excitement that I would love to capture and bottle for all teachers of mathematics.

Case 1. Seeing the Openness of Numbers

It was late December 2012, days before I was to fly to London for the holidays, when I first met Sebastian Thrun and his team at Udacity, a company producing online courses. I had been asked to visit Udacity to give the team advice about mathematics courses and ways to design effective learning opportunities. I walked into the airy space in Palo Alto that day and knew immediately that I had walked into a Silicon Valley startup—bikes were suspended on walls; young people, mostly men, wearing T-shirts and jeans, pored over computers or sat chatting about ideas; there were no office walls, only partitioned cubicles and light. I walked through the cubicles to the conference room at the back behind a glass wall. About 15 people had squeezed into the small room, sitting on chairs and the floor. Sebastian stepped forward and shook my hand, made some introductions, and invited me to sit down. He started firing questions at me: "What makes a good math course? How should we teach math? Why are students failing math?" He said that his friend Bill Gates had told him algebra was the reason we have widespread math failure in the United States. I cheekily replied, "Oh, Bill Gates the math educator told you that, did he?" His team members smiled, and Sebastian looked momentarily taken aback. He then asked, "Well, what do *you* think?" I told him that students were failing algebra not because algebra is so difficult, but because students don't have number sense, which is the foundation for algebra. Chris, one of the course designers who was also a former math teacher, nodded in agreement.

Sebastian continued firing questions at me. When he asked me what makes a good math question, I stopped the conversation and asked the group if I could ask *them* all a math question. They readily agreed, and I enacted a mini version of a number talk. I asked everyone to think about the answer to 18×5 and to show me, with a silent thumbs-up, when they had an answer. The thumbs started to pop up, and the team shared methods. There were at least six different methods shared that day, which I drew, visually, on the write-on table we sat around (see Figure 5.1).

We then discussed the ways the different methods were similar and different. As I drew the visual methods, the team members' eyes grew wider and wider. Some of them started to hop in their seats with excitement. Some shared that they had never imagined that there were so many ways to think about an abstract number problem; others said they were amazed that there was a visual image and it showed so much of the mathematics, so clearly.

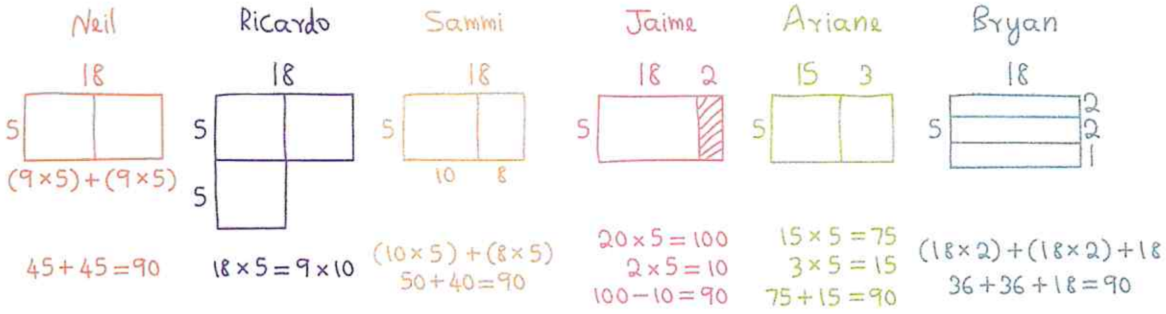


FIGURE 5.1 Visual solutions to 18 × 5

When I arrived in London, a few days later, I opened an email from Andy, the innovative young course designer at Udacity. He had made a mini online course on 18 × 5, which included going out on the street and interviewing passersby, collecting different methods. The team had been so excited by the ideas that they wanted to immediately put them out to the public, and they talked about making 18 × 5 T-shirts for everyone at Udacity to wear.

In the months following the meeting at Udacity, I met Luc Barthelet, then a director of Wolfram Alpha, one of the most important mathematics companies in the world. Luc had read about the different solutions to 18 × 5 I had shown in my book (Boaler, 2015a), which so excited him that he started asking everyone he met to solve 18 × 5. These reactions, these moments of intense mathematics excitement around an abstract number problem, seem important to share. How is it possible that these high-level math users, as well as young children, are so engaged by seeing and thinking about the different methods people use to solve a seemingly unexciting problem like 18 × 5? I propose that this engagement comes from people seeing the creativity in math and the different ways people *see* mathematical ideas. This is intrinsically interesting, but it's also true that most people I meet, even high-level mathematics users, have never realized numbers can be so open and number problems can be solved in so many ways. When this realization is combined with visual insights into the mathematical ways of working, engagement is intensified.

I have used this and similar problems with middle school students, Stanford undergrads, and CEOs of companies, all with equal engagement. I have learned through this that people are fascinated by flexibility and openness in mathematics. Mathematics is a subject that allows for precise thinking, but when that precise thinking is combined with creativity, flexibility, and multiplicity of ideas, the mathematics comes alive for people. Teachers can create such mathematical excitement in classrooms, with any task, by asking students for the different ways they see and can solve tasks and by encouraging discussion of different ways of seeing problems. In classrooms, teachers have to pay attention to classroom norms and teach students to listen to and respect each other's thinking; Chapter Seven will show a teaching strategy for this. When students have learned norms of respect and listening, it is incredible to see their engagement when different ways to solve a problem are shared.

Case 2. Growing Shapes: The Power of Visualization

The next case I want to share comes from a very different setting—a middle school classroom in a San Francisco Bay Area summer school where students had been referred because they were not performing well in the school year. I was teaching one of the four math classes with my graduate students at Stanford. We had decided to focus the classes on algebra, but not algebra as an end point, with students mindlessly solving for x. Instead, we taught algebra as a problem-solving tool that could be used to solve rich, engaging problems. The students had just finished sixth and seventh grades, and most of them hated math. Approximately half the students had received a D or an F in their previous school year (for more detail, see Boaler, 2015a; Boaler & Sengupta-Irving, 2015).

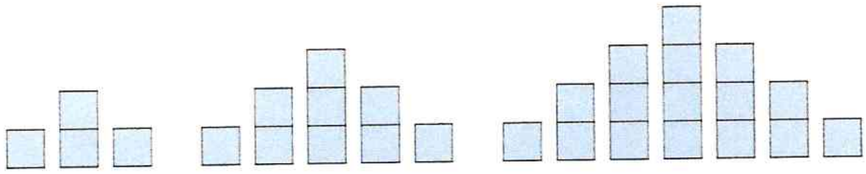
In developing a curriculum for the summer school, we drew upon a range of resources, including Mark Driscoll's books, Ruth Parker's mathematics problems, and two curricula from England—SMILE (which stands for secondary mathematics individualized learning experience) and Points of Departure. The task that created this case of mathematics excitement came from Ruth Parker; it asked the students to extend the growing pattern shown in Exhibit 5.1, made out of multilink cubes, to find how many cubes there would be in the 100th case. (Full-page task worksheets of all exhibits can be found in the Appendix.)

The students had multilink cubes to work with. In our teaching we invited the students to work together in groups to discuss ideas, sometimes groups we teachers chose, other times groups the students chose. On the day in question, I noticed an interesting grouping of three boys—three of the naughtiest boys in my class! They did not know each other before coming to the summer school, but all three spent most of the first week of summer school either off task or working to pull others off task. The boys would shout things out when others were at the board showing math and generally seemed more interested in social conversations than math conversations in the early days. Jorge had received an F in his last math class, Carlos a D, and Luke an A. But the day we gave the students this task, something changed. Incredibly, the three boys worked on this math task for 70 minutes, without ever stopping, becoming distracted, or moving off task. At one point some girls came over and poked them with pencils, which caused the boys to pick up their work and move to another table, they were so intensely engaged in the task and working to solve the problem.

All of our lessons were videotaped, and when we reviewed the film of the boys working that day we watched a rich conversation about number patterns, visual growth, and algebraic generalization. Part of the reason for the boys' intense engagement was an adaptation to the task that we had used—an adaptation that can be used with any math task. In classrooms, typically when function tasks such as the one we gave to the students are assigned, they are usually given with the instruction to find the 100th case (or some other high number) and ultimately the nth case. We did not start with this. Instead, we asked the students to first think alone, before moving to group work, about the ways they *saw* the shape growing. We encouraged them to think visually, not with numbers, and to sketch in their journals, showing us where they saw the extra cubes in each case. The boys saw the growth of the shape in different ways. Luke and Jorge saw the growth as cubes added to the bottom of the shape each time; this later became known by the class as

Shapes Task

How do you see the shapes growing?



Source: From Ruth Parker; a task used in MEC courses.

Exhibit 5.1

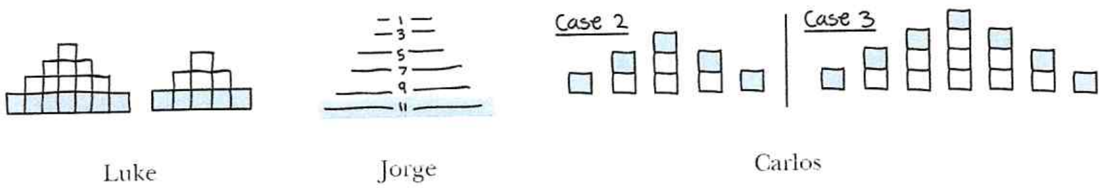


FIGURE 5.2 Students' work
Source: Selling, 2015.

the "bowling alley method," as the cubes arrived like a new line of pins in a bowling alley. Carlos saw the growth as cubes added to the top of the columns—what became known as the "raindrop method"—cubes dropping down from the sky, like raindrops, onto the columns (see Figure 5.2).

After the boys had spent time working out the function growth individually, they shared with each other their ideas for how the shape was growing, talking about where they saw the additional cubes in each case. Impressively, they connected their visual methods with the numbers in the shapes, not only working with their own methods but taking the time to explain the different methods to each other and using each other's methods. The three boys were intrigued by the function growth and worked hard to think about the 100th case, armed with their knowledge of the visual growth of the shape. They proposed ideas to each other, leaning across the table and pointing to their journal sketches. As is typical for mathematical problem solving, they zigzagged around, moving close to a solution, then further away, then back toward it again (Lakatos, 1976). They tried different pathways to the solution, and they broadly explored the mathematical terrain.

I have shown a video of the boys working to many conference audiences of teachers, and all have been highly impressed with the boys' motivation, perseverance, and high-level mathematical conversation. Teachers know that the perseverance shown by the three boys and the respectful ways they discussed each others' ideas, particularly in the context of summer school, is highly unusual, and they are curious as to how we were able to bring it about. They know that many students, particularly those who have been unsuccessful, give up when a task is hard and they don't immediately know the answer. That didn't happen in this case; when the boys were stuck, they looked back at their diagrams and tried out ideas with each other, many of which were incorrect but helpful in ultimately forming a pathway to the solution. After watching the case with teachers at conferences, I ask them what they see in the boys' interactions that could help us understand their high level of perseverance and engagement. Here are some important observations that reveal opportunities to improve the engagement of all students:

1) **The task is challenging but accessible.** All three boys could access the task, but it provided a challenge for them. It was at the perfect level for their thinking. It is very hard to find tasks that are perfect for all students, but when we open tasks and make them broader—when we make them what I refer to as "low floor, high ceiling"—this becomes possible for all students. The floor is low because anyone can see how the shape is growing, but the ceiling is high—the function the boys were exploring is a quadratic function whereby case n can be represented by $(n+1)^2$ blocks. We made the floor of the task lower by inviting the students to think visually about the case—although, as I will discuss later, this was not the only reason for this important adaptation.

2) **The boys saw the task as a puzzle**, they were curious about the solution, and they wanted to solve it. The question was not "real world" or about their lives, but it completely engaged them. This is the power of abstract mathematics when it involves open thinking and connection making.

3) **The visual thinking about the growth of the task gave the boys understanding of the way the pattern grew.** The boys could see that the task grew as a square of (n+1) side length because of their visual exploration of the pattern growth. They were working to find a complex solution, but they were confident in doing so, as they had been given visual understanding to help them.

4) The boys were encouraged by the fact that **they had all developed their own way of seeing the pattern growth** and their different methods were valid and added different insights into the solution. The boys were excited to share their thinking with each other and use their own and each other's ideas in the solving of the problem.

5) **The classroom had been set up to encourage students to propose ideas without being afraid of making mistakes.** This enabled the boys to keep going when they were "stuck," by providing ideas, right or wrong, that enabled the conversation to continue.

6) We had taught the students to **respect each other's thinking.** We did this by valuing the breadth of thinking everyone could offer, not just the procedural thinking that some could offer, valuing the different ways people saw problems and made connections.

7) The students were **using their own ideas,** not just following a method from a book as they learned core algebraic content. The fact that they had proposed different visual ideas for the growth of the function made them more invested and interested in the task.

8) **The boys were working together**; the video shows clearly the way the boys built understanding through the different ideas they shared in conversation, which also enhanced their enjoyment of the mathematics.

In all of these six cases of mathematics excitement, the mathematics task was central (and supported by important teaching). The next section will review the important design aspects of these six tasks that can be applied to all mathematics tasks, regardless of grade level. It is also important to note that in all six cases students were working with each other, sometimes thinking alone but often collaborating on ideas, in classrooms where they were given positive growth mindset messages. I will now turn to the ways we can build these important design elements into any mathematics task.

From Cases of Mathematics Excitement to the Design of Tasks

We are emerging from an unproductive period in education. Since the No Child Left Behind Act was introduced by the Bush administration, teachers started to come under pressure to use "scripted" curriculum and pacing guides, even though they knew they were damaging their students. Many teachers felt deprofessionalized by this; they felt that important teaching decisions had been taken out of their hands. Fortunately, this time is ending; we are entering a much more positive time, with teachers being trusted to make important professional decisions. One of the aspects of teaching for a mathematics mindset that I am most excited about is the transformations that we can make in mathematics classrooms through giving important messages and opening up mathematics tasks. This opening up of tasks gives students the space to learn and is absolutely essential in developing mathematical mindsets.

A range of rich, open tasks are now available to teachers through websites, which I will list at the end of this chapter. But many teachers do not have time to search through websites.

Fortunately, teachers don't need to find new curriculum materials, as they can make adaptations to the tasks in the curriculum they use, opening them to create new and better opportunities for students. To do this, teachers may need to develop their own new mindsets as designers—that is, as people who can introduce a new idea and create new, enhanced learning experiences. The mathematical excitement I described earlier came, in a number of cases, from adapting a familiar task. In the growing shapes task, for example, the simple instruction for students to visualize the shapes growing changed everything, giving students access to understandings that would not have been possible otherwise. When teachers are designers, creating and adapting tasks, they are the most powerful teachers they can be. Any teacher can do this; it does not require special training. It involves knowing about the qualities of positive math tasks and approaching tasks with the mindset to improve them.

In designing and adapting math tasks for better learning, there are six questions that, if asked and acted upon in the task, increase their power incredibly. Some tasks are more suited to some questions than others, and many are naturally combined, but I am confident in saying every task will be made richer by paying attention to at least one of the following six questions.

1. Can You Open the Task to Encourage Multiple Methods, Pathways, and Representations?

There is nothing more important that teachers can do with tasks than to open them up so students are encouraged to think about different methods, pathways, and representations. When we open a task we transform its learning potential. Opening can happen in many ways. Adding a visual requirement, such as those shown in the growing shapes and negative space tasks, is a great strategy. Another way to open a task that is extremely mathematically productive is to ask students to make sense of their solutions.

Cathy Humphreys is wonderful teacher. In a book we coauthored, we show six video cases of Cathy teaching her seventh-grade class, accompanied by her lesson plans. One of the videos shows Cathy asking the students to solve: 1 divided by 2/3. This could be a closed, fixed mindset question with one right answer and one method, but Cathy transforms the task by adding two requirements: that students make sense of their solution and that they offer a visual proof (see Figure 5.19). She starts the lesson by saying "You may know a rule for solving this question, but the rule doesn't matter today, I want you to make sense of your answer, to explain why your solution *makes sense*."

In the video case we see that some students thought the answer was 6, because you can manipulate the set of numbers (1, 2, and 3), with no mathematical sense making, and make 6. But they struggled to show this visually or make sense of it. Others were able to show, in a range of different visual representations, why there were one and a half 2/3's "inside 1." The requirement for students to show their thinking visually and make sense of their answers transformed the question from a fixed to a growth mindset task, and created a wonderful lesson, filled with sense making and understanding.

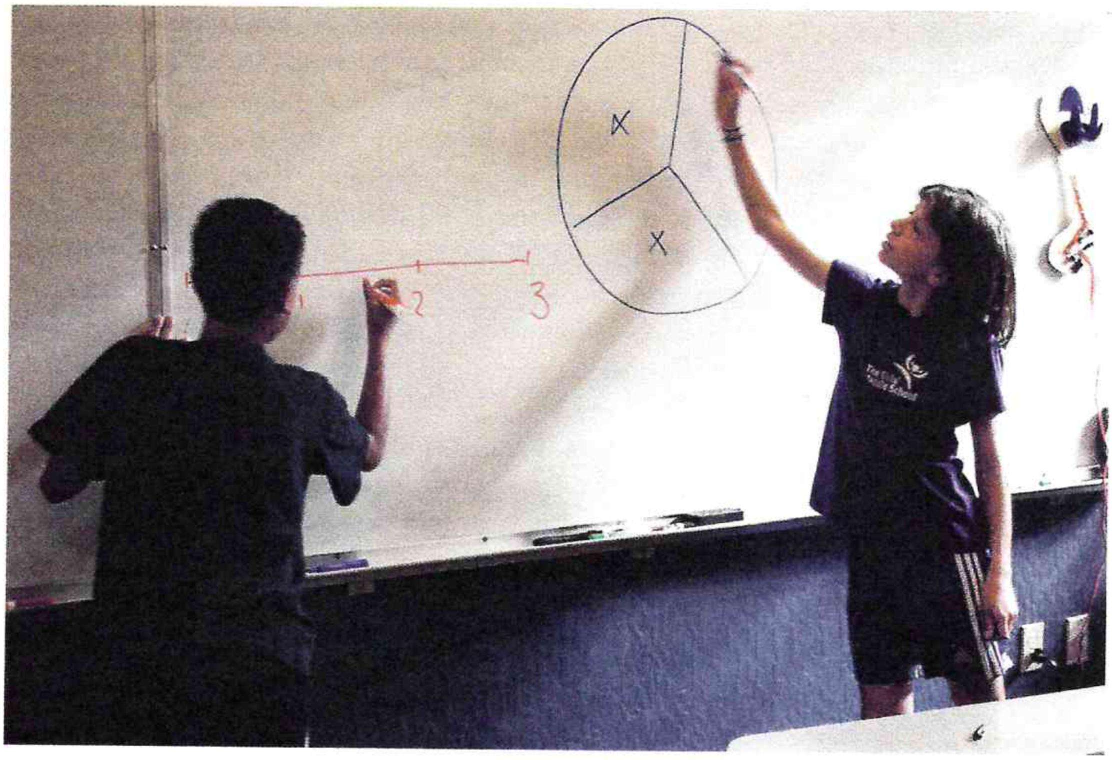


FIGURE 5.19 Students share solutions to 1 ÷ 2/3

2. Can You Make It an Inquiry Task?

When students think their role is not to reproduce a method but to come up with an idea, everything changes (Duckworth, 1991). The same mathematics content can be taught with questions that ask for a procedure or as questions that ask for students to think about ideas and use a procedure. For example, instead of asking students to find the area of a 12 by 4 rectangle, ask them how many rectangles they can find with an area of 24. This small adaptation changes students' motivation and understanding. In the inquiry version of the task, students use the formula for the area of a rectangle, but they also need to think about spatial dimensions and relationships, and what happens when one dimension changes (see Figure 5.20). The mathematics is more complex and exciting because students are using their ideas and thoughts.

Instead of asking students to name quadrilaterals with different qualities, ask them to come up with their own, as shown in Exhibit 5.5.

Another excellent task is the four 4's (see Exhibit 5.6). In this task you ask students to make all the numbers between 1 and 20 using four 4's and any operation; for example:

$$\sqrt{4} + \sqrt{4} + 4/4 = 5$$

This is a great activity for practicing operations, but it does not look like a practicing operations task, because the operations are beautifully embedded inside an inquiry task. When we posted

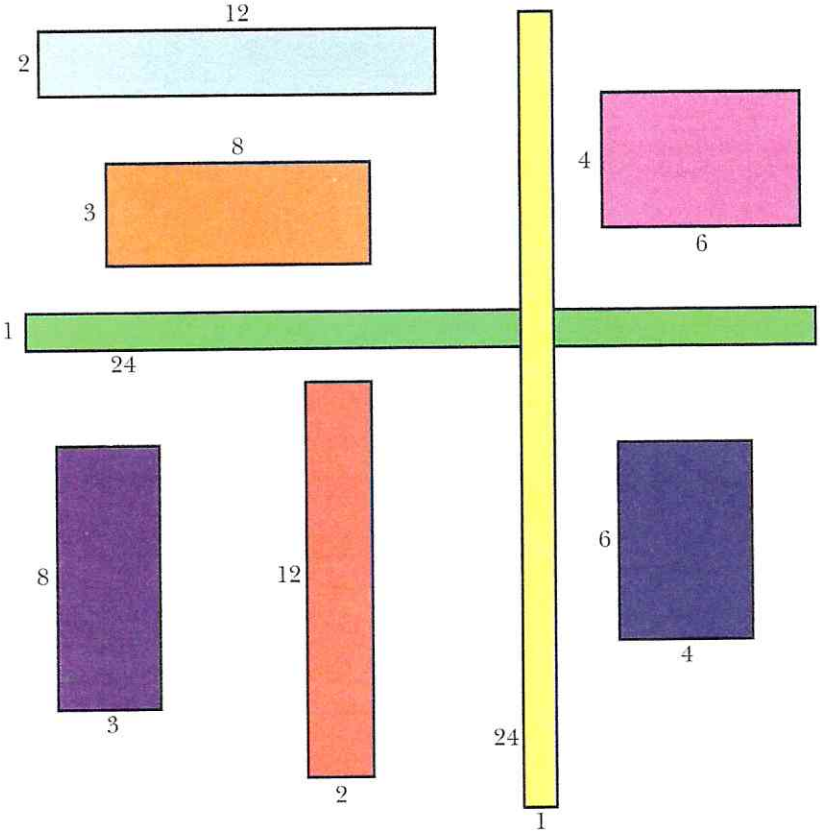


FIGURE 5.20 Rectangles with an area of 24

Find Quadrilaterals!

Pairs of Equal Sides \ Pairs of Parallel Sides	0	1	2
0			
1			
2			

Exhibit 5.5

Four 4's

Can you find every number between 1 and 20 using only four 4's and any operation?

Going beyond …

Can you find more than one way to make each number with four 4's?

Can you go beyond 20?

Can you use four 4's to find negative integers?

Exhibit 5.6

this task on Youcubed.org, teachers told us the task was incredible. Here are two comments from Youcubed teachers:

> "My students were so inspired &excited with the four 4's they decided to investigate three 3's, and the sky was the limit."
>
> "The fours problem was amazing! I used it in my sixth-grade math class, and students were creating equations that led to discussions about distributive property, order of operations, variables … it was fantastic!"

(The full task on Youcubed includes advice on ways to introduce the task and organize students; see https://www.youcubed.org/wim-day-1/.)

Another way to open a task and make it an inquiry task is to ask students to write a magazine article, a newsletter, or a short book about it. This structure can work with any content. At Railside, in ninth grade the students were asked to write a book on $y = mx + b$; they filled pages showing what this meant, how it could look visually, situations in which it could be used, and their ideas on the meaning of the equation. In a high school geometry unit that three of my graduate students at Stanford (Dan Meyer, Sarah Kate Selling and Kathy Sun) created with me, we asked students to write a newsletter on similarity, using photos, tasks, cartoons, and any other media they wanted to show what they knew about the topic (see https://www.youcubed.org/wp-content/uploads/The-Sunblocker1.pdf). Exhibit 5.7 is a general form of the newsletter assignment we gave out.

3. Can You Ask the Problem Before Teaching the Method?

When we pose problems for which students need to know a method before we introduce the method, we offer a great opportunity for learning and for using intuition. The tasks described earlier that exemplified this were the finding the largest enclosure area for a fence task and the finding the volume of a lemon task. But this design component can be used with any area of mathematics—in particular, for any teaching of a standard method or formula, such as the area of shapes, the teaching of pi, and statistical formulas such as mean, mode, range, and standard deviation. Exhibit 5.8 shows an example.

After students have worked out their own ways of finding averages and discussed them as groups and as a class, they could be taught the formal methods of mean, mode, and range.

4. Can You Add a Visual Component?

Visual understanding is incredibly powerful for students, adding a whole new level of understanding, as we saw in the growing shapes task. This can be provided through diagrams but also through physical objects, such as multilink cubes and algebra tiles. I spent my early years growing up with Cuisenaire rods, as my mother was training to be an elementary teacher. I spent many happy hours playing with the rods, ordering them and investigating mathematical patterns. In an online course designed to give students important mathematics strategies, I teach students to draw any mathematics problem or idea (see https://class.stanford.edu/courses/Education/EDUC115-S/Spring2014/about). Drawing is a powerful tool for mathematicians and mathematical problem solvers, most of whom draw any problem they are given. When students are stuck in math class, I often ask them to draw the problem out.

The Long Jump

You are going to try out for the long jump team, for which you need an average jump of 5.2 meters. The coach says she will look at your best jump each day of the week and average them out. These are the five jumps you recorded that week:

	Meters
Monday	5.2
Tuesday	5.2
Wednesday	5.3
Thursday	5.4
Friday	4.4

Unfortunately, Friday's was a low score because you weren't feeling that well!

How could you work out an average that you think would fairly represent your jumping? Work out some averages in different ways and see which you think is most fair, then give an argument for why you think it is fairest. Explain your method and try and convince someone that your approach is best.

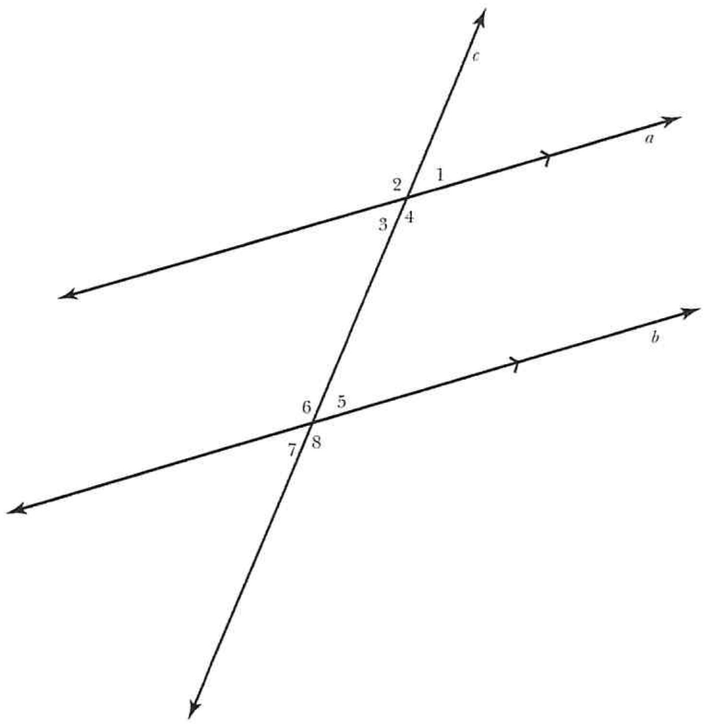
Exhibit 5.8

Railside School, the highly successful school I studied, asked students to show connections through color coding. For example, when teaching algebra, they asked students to show functional relationships in many forms: as an expression, as a picture, in words, and on a graph. Many schools ask for these different representations. Railside was unusual in that they asked the students to show relationships in color; for example, to show the x in the same color in an expression, on the graph, and in the diagram. Chapter Seven, which describes the Railside approach in more detail, shows one of their color coding tasks. In other topic areas—for example, when asking students to identify congruent, vertical, and supplementary angles—you could also ask them to color and write about as many relationships as they can, using color to highlight the relationships. Exhibit 5.9 and Figure 5.21 show an example.

Further examples of color coding are given in Chapter Nine.

Parallel Lines and a Transversal

1. Use color coding to identify congruent angles.
2. Identify vertical and supplementary angles.
3. Write about the relationships you see. Use the color from your diagram in your writing.



Vertical Angles:

Supplementary Angles:

Relationships:

Exhibit 5.9

5. Can You Make It Low Floor and High Ceiling?

All of the preceding problems are low floor and high ceiling. The breadth of the space inside them means that they are accessible to a wide range of students and they extend to high levels.

One way to make the floor lower is to always ask students how they see a problem. This is an excellent question for other reasons too, as I have explained.

A great strategy for making a task higher ceiling is to ask students who have finished a question to write a new question that is similar but more difficult. When we were teaching a group of

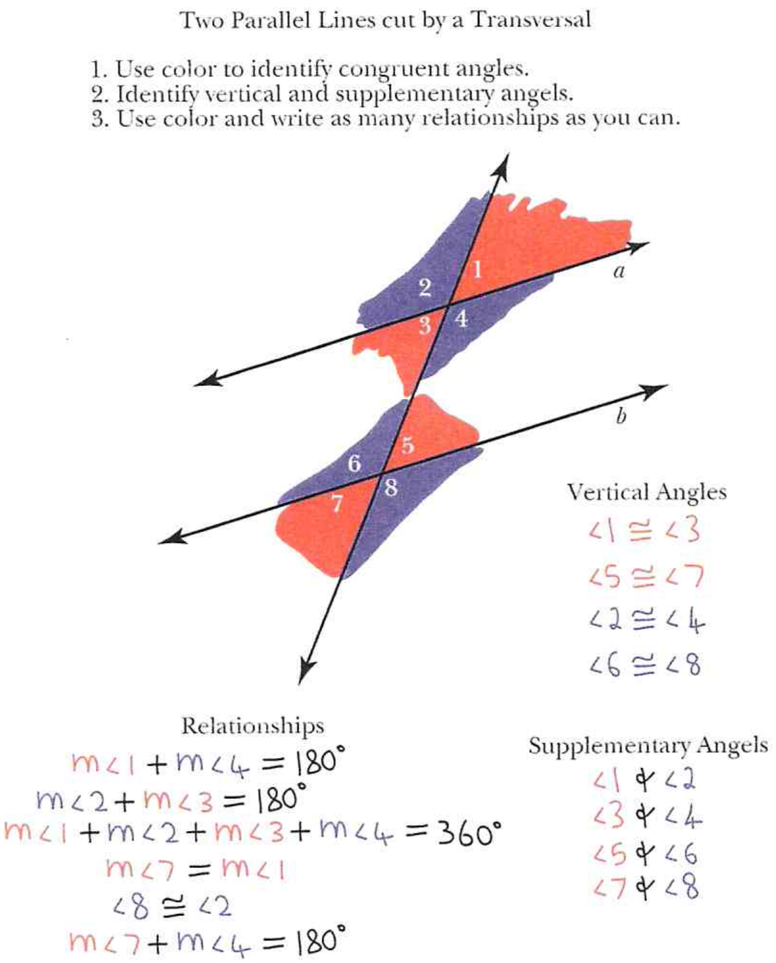


FIGURE 5.21 Color coding angles

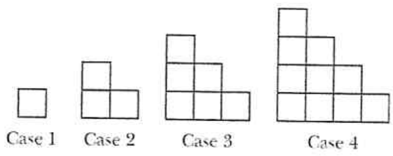
heterogeneous students in summer school, we used this strategy a lot to great effect. For example, when one student, Alonzo, finished the staircase task, which asked students to think about pattern growth and the nth case (see Exhibit 5.10), he asked a harder question. He asked how a staircase extending in four directions would grow and the number of cubes in the nth case (see Figure 5.22).

When students are invited to ask a harder question, they often light up, totally engaged by the opportunity to use their own thinking and creativity. This is an easy extension for teachers to use and one that they can have available in any lesson. With any set of mathematics questions, consider giving students a task like this:

> "Now you write a question; try to make it hard ☺"

Students can give their questions to other students, who can be encouraged to write questions for each other. This is a particularly good strategy to use for students who work faster than other students or who complain that work is too easy for them, as it involves deep and difficult thinking.

Staircase



How do you see the pattern growing?

How many would be in the 100th case?

What about the nth case?

Exhibit 5.10

6. Can You Add the Requirement to Convince and Reason?

Reasoning is at the heart of mathematics. When students offer reasons and critique the reasoning of others, they are being inherently mathematical and preparing for the high-tech world they will be working in, as well as the Common Core. Reasoning also gives students access to understanding. In my four-year study of different schools, we found that reasoning had a particular role to play in the promotion of equity, as it helped to reduce the gap between students who understood and students who were struggling. In every math conversation, students were asked to reason, explaining why they had chosen particular methods and why they made sense. This opened up mathematical pathways and allowed students who had not understood to both gain understanding and ask questions, adding to the understanding of the original student.

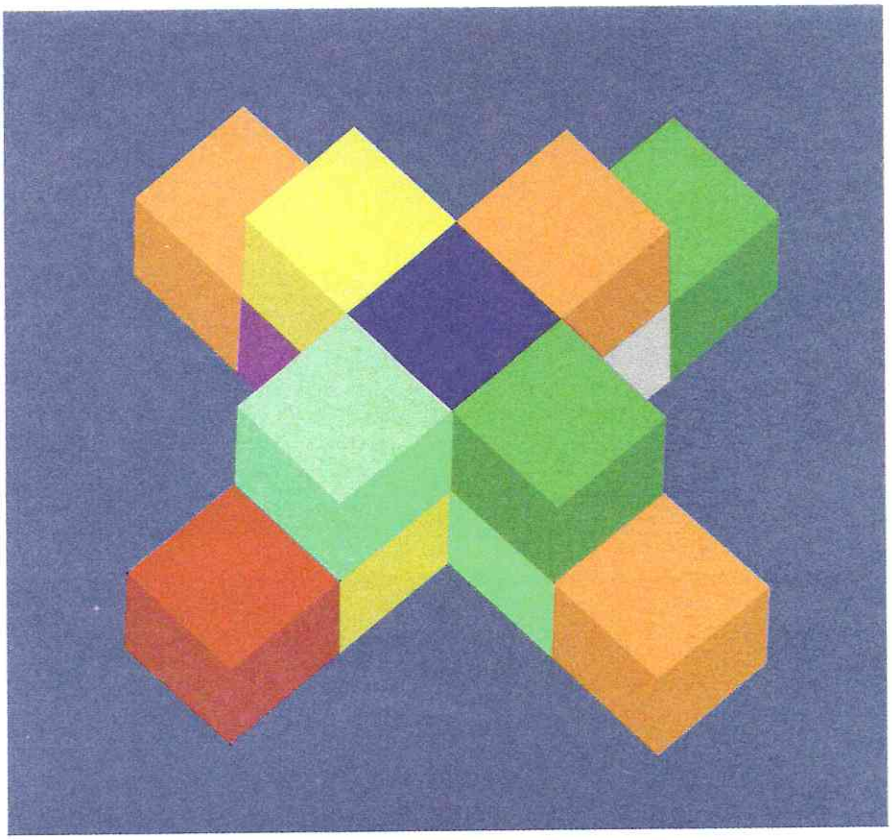


FIGURE 5.22 Alonzo's extension problem

I like to accompany one of my favorite tasks for encouraging reasoning with a pedagogical strategy that has many benefits. I learned this strategy from Cathy Humphreys, who asks her students to be skeptics. She explains that there are three levels of being convincing (Boaler & Humphreys, 2005):

Convince yourself

Convince a friend

Convince a skeptic

It is fairly easy to convince yourself or a friend, but you need high levels of reasoning to convince a skeptic. Cathy tells her students that they need to be skeptics, pushing other students to always give full and convincing reasons.

A perfect task to teach and encourage higher levels of reasoning that can be accompanied by the skeptic role was developed by Mark Driscoll; it is called "paper folding." I have used this task with a range of different groups, always with very high levels of engagement. Teachers tell me they love this task, as it often lets students shine who don't typically get that opportunity. In this task, students work in pairs with a square piece of paper. They are asked to fold the paper to make new shapes. Exhibit 5.11 shows the five, progressively more challenging questions (see Figure 5.23).

Paper Folding

Work with a partner. Take turns being the skeptic or the convincer. When you are the convincer, your job is to be convincing! Give reasons for all of your statements. Skeptics must be skeptical! Don't be easily convinced. Require reasons and justifications that make sense to you.

For each of the following problems, one person should make the shape and then be convincing. Your partner is the skeptic. When you move to the next question, switch roles.

Start with a square sheet of paper and make folds to construct a new shape. Then, explain how you know the shape you constructed has the specified area.

1. Construct a square with exactly 1/4 the area of the original square. Convince your partner that it is a square and has 1/4 of the area.

2. Construct a triangle with exactly 1/4 the area of the original square. Convince your partner that it has 1/4 of the area.

3. Construct another triangle, also with 1/4 the area, that is not congruent to the first one you constructed. Convince your partner that it has 1/4 of the area.

4. Construct a square with exactly 1/2 the area of the original square. Convince your partner that it is a square and has 1/2 of the area.

5. Construct another square, also with 1/2 the area, that is oriented differently from the one you constructed in 4. Convince your partner that it has 1/2 of the area.

Source: Adapted from Driscoll, 2007, p.90, http://heinemann.com/products/E01148.aspx

Exhibit 5.11

When I have given this task to teachers they have struggled for a long time on question 5, some working well into the evening after a full day of professional development, enjoying every moment. Their engagement is enhanced with having a physical shape to consider and change, but also by the need to be convincing. When I give students and teachers this task, I ask for the pairs to take turns, with one folding and convincing and one being the skeptic; then they switch for the next question. When I ask students to play the role of being the skeptic, I explain that they need to demand to be fully convinced. Students really enjoy challenging each other for convincing reasons, and this helps them learn mathematical reasoning and proof. As a teacher you may want to model what a fully convincing answer is, by asking students follow-up questions if they have not been convincing enough.

Another example of a task that involves convincing is shown in Exhibit 5.12. The request for students to reason and be convincing can be applied to any mathematics problem or task.

Conclusion

When mathematics tasks are opened for different ways of seeing, different methods and pathways, and different representations, everything changes.

Questions can move from being closed, fixed mindset math tasks to growth mindset math tasks, with space within them to learn. To summarize, these are my five suggestions that can work to open mathematics tasks and increase their potential for learning:

1. Open up the task so that there are multiple methods, pathways, and representations.
2. Include inquiry opportunities.
3. Ask the problem before teaching the method.
4. Add a visual component and ask students how they see the mathematics.
5. Extend the task to make it lower floor and higher ceiling.
6. Ask students to convince and reason; be skeptical.

Further examples of tasks with these design features are given in Chapter Nine.

If you take the opportunity to modify tasks in these ways, you will be offering your students more and deeper learning opportunities. I have really enjoyed all the times I have seen students working on rich open mathematics tasks, and I have taught with them myself, as students are so excited by them. They love to make connections, which are so important in mathematics, and visual, creative mathematics is inspiring to students. A week of mathematics lessons that include the design features discussed in this chapter, and that are appropriate for grades 3 to 9, can be viewed and downloaded freely here: https://www.youcubed.org/week-of-inspirational-math/.

When I trialed these lessons in middle school classrooms, I experienced parents rushing up to me to tell me that these lessons had changed mathematics for their children. Some parents told me that their children had always disliked math until they took these lessons and saw mathematics in a completely different light. With a design and mathematical mindset, teachers (and parents) can create and transform mathematics tasks, giving all students the rich mathematics environment that they deserve. We cannot wait for publishing companies to realize these changes are needed and make the necessary changes, but teachers can make these changes–creating open, engaging mathematics environments for all of their students.

The following websites provide mathematics tasks that incorporate one or more of the features I have highlighted:

- Youcubed: www.youcubed.org
- NCTM: www.nctm.org (membership required to access some of the resources)
- NCTM Illuminations: http://illuminations.ntcm.org
- Balanced Assessment: http://balancedassessment.concord.org
- Math Forum: www.mathforum.org
- Shell Center: http://map.mathshell.org/materials/index.php
- Dan Meyer's resources: http://blog.mrmeyer.com/
- Geogebra: http://geogebra.org/cms/
- Video Mosaic project: http://videomosaic.org/
- NRich: http://nrich.maths.org/
- Estimation 180: http://www.estimation180.com
- Visual Patterns; grades K–12: http://www.visualpatterns.org
- Number Strings: http://numberstrings.com
- Mathalicious, grades 6–12; real-world lessons for middle and high school: http://www.mathalicious.com