Designing and scaffolding rich mathematical learning experiences with challenging tasks

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In response to increasing teacher interest in how to design and implement effective challenging tasks, James presents the Launch/Explore/Discuss model that builds on the work of leading Australian educators and researchers Peter Sullivan, Doug Clarke, and Charles Lovitt.

In my capacity running professional learning sessions in schools I have, on occasion, had the privilege of being invited to step into another teacher's classroom, and lead a mathematics lesson. More often than not, these teachers are interested in how to teach with challenging tasks. This is testament to the power of teaching through problem solving, and the influence of Australian teacher-educators and researchers such as Peter Sullivan, Doug Clarke and Charles Lovitt (and many others) have had on shaping what is considered good practice when teaching primary mathematics.

This article describes my experience facilitating a challenging task in a Year 4 classroom focussed around exploring relationships between different linear counting patterns. The second part of the article briefly describes how this task could be expanded into a four-lesson mini-sequence of learning through drawing on principles of variation theory. However, before proceeding, it is perhaps necessary to briefly describe the research underpinning challenging tasks and their pedagogical characteristics.

Pedagogical characteristics of challenging tasks

Challenging tasks can be viewed as a subset of problem-solving tasks that possess particular design characteristics. Summarising the work of Peter Sullivan and colleagues (e.g., Sullivan et al., 2011, 2015; Sullivan & Mornane, 2014), at least four distinctive claims can be made about the design characteristics of challenging tasks.

1. As students are expected to plan their own approach to the task and contribute to a collaborative learning environment, it is critical that the task has multiple solution pathways and perhaps multiple solutions, both to support student agency and to allow for students to compare and contrast their approach with peers.
2. As this pedagogical approach is premised on the idea that students learn best when provided with opportunities to struggle and spend time in the 'zone of confusion', all students should experience at least some important aspects of the task as mathematically challenging.
3. As a single task has been designed to function as the main focus of the lesson, it needs to meaningfully engage students in important mathematical work for a substantial period of time (e.g., 20 minutes).
4. As all students are expected to participate in the lesson regardless of their demonstrated (or perceived) mathematical ability, the task is generally prepared with accompanying enabling and extending prompts.

Enabling and extending prompts are a technology to support differentiated learning experiences whilst allowing all students to learn mathematics through problem-solving (Sullivan, Mousley, & Jorgensen, 2009). Enabling prompts are designed to ensure a task is accessible to a larger range of learners through changing how the original problem is represented, helping the student connect the problem to prior learning, and/or removing a step in the problem. It is worth emphasising that following engagement with the enabling prompt, the general expectation is that the student will then return to the main task. Extending prompts, by contrast, are designed for

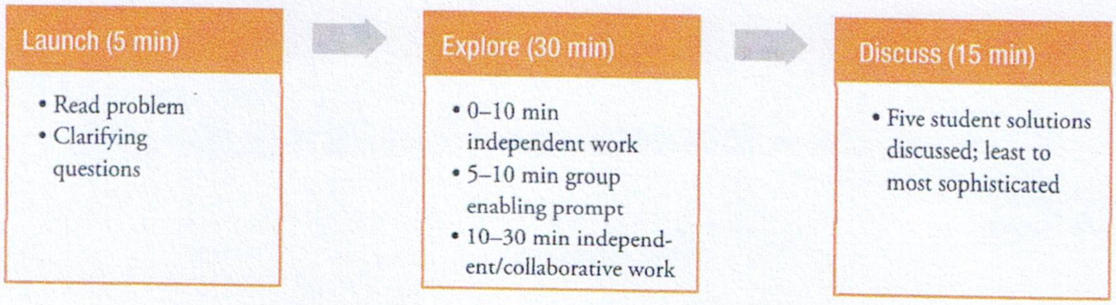

Figure 1. Overview of structure of lesson.

students who complete the original task. It exposes these students to an additional task that is more challenging; however that requires them to use similar mathematical reasoning, conceptualisations, and representations (Sullivan et al., 2009).

There is substantial evidence to support many of the assumptions underpinning the design characteristics of challenging tasks. For example, learning appears to be enhanced when students are provided with opportunities to construct their own approach to problems compared with being taught explicit procedures (Jonsson, Norqvist, Liljekvist, & Lithner, 2014). Such opportunities lead to larger gains in mathematical performance even when students do not arrive at a solution for a given task (Kapur, 2014). The power of challenging tasks is strongly connected to their capacity to support episodes of 'productive struggle' (Pasquale, 2016).

Productive struggle can be understood as a precondition for facilitating 'aha' moments; those moments where learning almost becomes tangible, and which tend to be treasured by teachers and students alike. In the lesson described in the next section, one particular Year 4 student Damon had an 'aha' moment that will likely stay with me for many years to come. As is hopefully clear from my elaboration, it was the structure of the task and my willingness to allow students to struggle, that, more than other factor, led to the positive learning experience for Damon and many of his classmates.

The lesson context

The lesson I am describing took place in a government-run primary school in regional Victoria, Australia in a Year 4 class (with 28 students). We had around 50 minutes for the session. The lesson was designed to follow the launch-explore-discuss (summarise) structure advocated by proponents of teaching through problem-solving (Lampert, 2001; see Figure 1).

In consultation with the classroom teacher beforehand, we had established the learning foci for the lesson. The primary learning focus was for students to explore the relationship between different linear counting sequences (i.e., different skip-counting patterns). In particular, students would be expected to distinguish between numbers that appear in many different sequences (i.e., numbers with many factors) and numbers that occur in few sequences (e.g., prime numbers). I value this learning focus because it makes it clear that lessons concerned with number patterns are not only about fluency, and that the proficiencies of understanding, reasoning and problem solving can be considered equally relevant. A secondary learning focus was for students to be able to recall fluently relevant counting sequences to solve a problem.

Before entering the classroom, I made sure I had all the materials I needed to support the lesson including dice (6-sided, 10-sided, 12-sided, 20-sided), number charts (covering numbers 1-100), as well as enabling and extending prompts.

Phase 1: Launch

The first phase of the lesson began with me posing the Lucky Dice task to students (see textbox), displaying the task on the electronic whiteboard and reading it aloud. Students were invited to ask any clarifying questions. They were then instructed that they would work on the task independently, without any support, for five minutes. Students were told that they could have access to a number chart, and a 6-sided dice, if they thought these would be useful.

> **The task: Lucky Dice (Lesson 1)**
> My dad offered me a deal. I choose any number on a hundreds chart. He'd then roll a 6-sided dice, and we'd count by whatever number he rolled (from zero). If we land on my number, he'd give me 10 dollars. If we skip my number, I'd give him 10 dollars. What are some good numbers I could choose? Should I take the deal?

Phase 2: Explore

I set a countdown timer for five minutes in a manner that was visible to students (who were used to this routine—although not in a mathematics class focussed on problem solving), and proceeded to roam around the classroom to gauge how students were approaching the problem. The purpose of giving them time to work on the problem independently before collaborating, or seeking support, was to allow students to gauge their own understanding of the problem, and to consider what relevant prior learning they may be able to draw on. Importantly, this five minutes is likely to be a frustrating time for many students. This is intentional. We are hoping that students will enter what has been termed the "zone of confusion" (Clarke, Cheeseman, Roche & Van Der Schans, 2014, p. 58). It is based on the assumption that getting students to expend effort "in order to make sense of mathematics, to figure out something that is not immediately apparent", is integral to the learning experience (Hiebert & Grouws, 2007, p. 387). As students become more used to experiencing mathematics taught in this manner, I might look to extend this initial 'independent work time' following the launch phase to, for example, 10 minutes (depending on the task at hand, and the age group of the students).

After five minutes had elapsed, I instructed students that anyone struggling to make progress could join me on the floor where I would share with them the first 'hint sheet' (the first enabling prompt). Ideally, students access enabling prompts of their own volition independently of the teacher (Russo, 2018), however on this occasion it was appropriate to introduce the prompt to a small group of students to clarify any ongoing confusion regarding the actual mechanics of the task. This was particularly the case because this cohort of students were not used to tasks being posed in this manner, and were accustomed to more teacher explanation than I had provided. Around ten or so students joined me on the floor, with a few others listening from their tables. I read out the first enabling prompt (see textbox).

> **Enabling prompt 1**
>
> What if I chose the number 13? Roll the dice 10 times, and see how many times I win the bet. Can you choose a better number than 13?

The group gathered on the floor in front of an interactive number chart (Splat Square) on our electronic whiteboard. I chose the number 13, marked it on the chart, and students who had joined me took turns rolling the dice. We used the interactive number chart to keep track of our counting patterns, and I recorded our results. We 'won' three times (we rolled three ones) and lost seven times, meaning that we had to pay dad $40. I left the students with the prompt 'Can you choose a better number than 13?'.

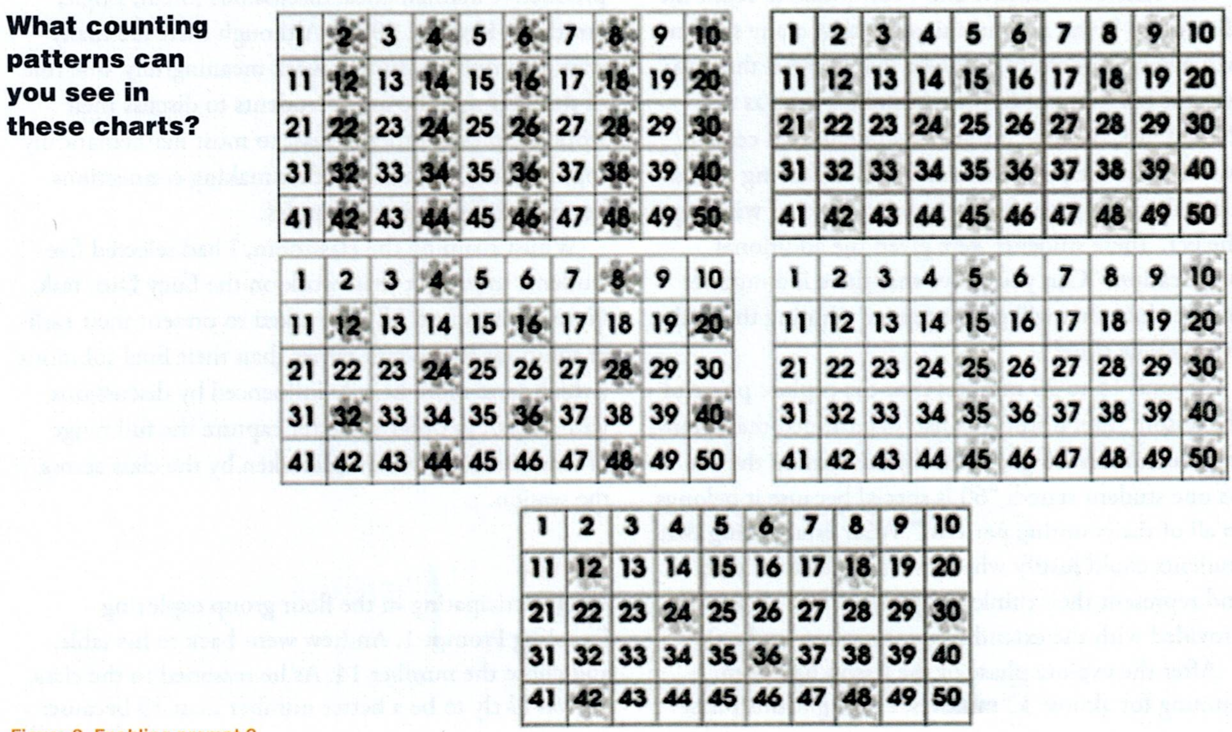

Figure 2. Enabling prompt 2.

The first enabling prompt had taken me around five minutes to administer in the small group setting, meaning that students had now spent 10 minutes in the explore phase of the lesson. I stopped the class and let them know that a second 'hint sheet' (Enabling Prompt 2) was available at the front of the classroom should they desire it (see Figure 2). I also let students know that at this stage, they could choose to work collaboratively with other students in groups of two or three. A majority of students began to work in loose groups, with several students moving between undertaking their independent investigations, and sharing their thinking and findings with a partner.

It is worth noting that Enabling Prompt 2 is essentially sacrificing our secondary learning objective (i.e., for students to be able to recall fluently relevant counting sequences to solve a problem) in order to allow students to focus on the primary learning objective (i.e., for students to explore the relationship between different linear counting sequences). Prompts have the power to direct and shape the learning experience, and should serve to focus student attention toward the key idea inherent in the lesson. Consequently, it is vital that a teacher has given at least some thought to a potential hierarchy of intended learning objectives when choosing or developing prompts during the lesson-planning phase (Russo & Hopkins, 2017). During the remainder of the explore phase, around half a dozen or so students accessed this second enabling prompt.

The classroom teacher and I continued to roam the classroom. It was around this stage that many students had begun to identify numbers that allowed them to win the bet 5 out of 6 times (numbers such as 12, 24 and 30). *In some instances, students had ceased* working on the problem, satisfied with having found a number that gave them a 'good chance' of winning the bet. These students were given the additional provocation: "Can you prove that there is a number on the chart that will guarantee me winning the deal every single time?".

Around 15 to 20 minutes into the explore phase of the lesson, several students had determined that choosing the number 60 guaranteed you winning the bet. As one student stated: "60 is special because it belongs to all of the counting patterns". After establishing that students could justify why this was in fact the case, and represent their thinking appropriately, they were provided with the extending prompt (see textbox).

After the explore phase of the lesson had been running for almost 30 minutes, I brought this phase to the end. Students were invited to bring their work to the floor to discuss their solutions.

Extending prompt

What about if dad rolled a 10-sided dice instead, but let me choose any number up to 1000? How would this change the number I might choose? Am I still guaranteed of winning the deal?
And for an extra challenge: Assuming dad rolled a 10-sided dice, what is the smallest number I can choose that guarantees me winning the deal?

Phase 3: Summarise and discuss

During the explore phase, one of the responsibilities of the teacher whilst monitoring student work on the task is to select several students to present their work to the class, and think about how these student responses should be sequenced. The teacher also has to consider the mathematical connections both between different student solutions to the task, and the underlying concepts that are the focus of the lesson. This process is supported by teachers anticipating potential student responses ahead of time, which is in turn supported by them working through the task themselves in their planning teams. Collectively, these notions of anticipating, monitoring, selecting, sequencing and connecting are referred to as five practices for orchestrating productive mathematical discussions (Stein, Engle, Smith, & Hughes, 2008). Although there are many ways to sequence student work meaningfully, one rule of thumb might be to get students to discuss their work samples in order of least to most mathematically sophisticated, with the teacher making connections between different work samples.

Whilst roaming the classroom, I had selected five students to present their work on the Lucy Dice task. Note Andrew and Jill were asked to present their earlier solutions to the task rather than their final solutions (which were more heavily influenced by discussions with peers), in order to better capture the full range of mathematical work undertaken by the class across the session.

Andrew

After participating in the floor group exploring Enabling Prompt 1, Andrew went back to his table, and chose the number 14. As he reasoned to the class, 14 was likely to be a better number than 13 because:

"All even numbers are pretty lucky, because if we roll a 2 we land on an even number, and if we roll a 1, we land on an even number. For odd numbers, we'd have to roll a 1". Andrew had proceeded to circle all of the even numbers on his chart, and declared these lucky numbers. During our discussion, Peta challenged Andrew: "But what about 9? We can land on 9 if we roll a 3?".

I asked the class if they could think of any other examples of odd numbers that might be "lucky"? Tony offered 15, which was a number he himself had considered "because 5 times 3 is 15"; however, he then said that he thought 12 was "luckier" than 15. I closed this part of the discussion by asking the group a question simultaneously intended to partially validate Andrew's thinking, and to consolidate our discussion so far: "Do we agree with Andrew that even numbers tend to belong to more counting patterns than odd numbers?". The class concurred, and Jill (the next student I had nominated to share their thinking) was asked whether she had also decided on an even number.

Jill

Jill chose the number 100 as her final number; however, on her chart, she had marked all of the multiples of ten. She offered: "All numbers ending with zero are lucky – because they are part of the counting by 5s pattern, and the counting by 2s pattern. They're good numbers to choose". In a similar manner to Andrew, Jill had put forward an explanation to justify the utility of a set of numbers (rather than one specific number), which is one of the reasons I asked her to contribute her work. However, Jill had gone beyond Andrew's thinking by relating her explanation more directly and comprehensively to counting patterns (rather than even and odd numbers); although it was clear that she had not thought much about the fact that some numbers ending in zero belong to more counting patterns than others. I was conscious that many students had also chosen multiples of 10 as their number (in particular 30 and 60), and had employed more sophisticated reasoning than Jill. Rather than let the discussion evolve to this next level, I wanted to first engage Damon's thinking. I cut the discussion of Jill's work sample short, and posed the question: "Jill thought that tens numbers were good to choose. I wonder if anyone decided that a number less than 10 could help us win this bet?".

Damon

Damon was a student described by his teacher as someone who found mathematics very difficult and who was generally disengaged in mathematics lessons. However, after participating in the small group that explored Enabling Prompt 1, Damon had returned to his table with his dice, determined to find a better number than 13. Damon had proceeded to play against his dad for much of the next 20 minutes of the session, simulating the bet by rolling his dice. He had explored a couple of different numbers before settling on the number 6. In his words: "6 is special because it is means I have 4 ways of winning the bet – 1,2,3 and 6". Damon had 'won' $230 after choosing the number 6, and was visibly excited by the fact that he was 'beating his dad'. Despite having difficulty with many basic skip-counting sequences (e.g, counting by 3s fluently beyond 12), Damon had still managed to engage in the primary learning focus of the lesson through accessing enabling prompts. The next question I put to the class was: "Did anyone come up with an even better number than 6?".

Jennifer

Jennifer was the next student to share her thinking. Like several students, Jennifer had approached the task methodically, using her hundreds chart to systematically count by 1s, 2s, 3s, 4s, 5s and 6s, recording each counting sequence in a different colour. She indicated that "60 is the only number I landed on 6 times", and several students concurred that 60 was indeed the number that guaranteed me winning the bet. The final student asked to share his thinking was Max.

Max

Max indicated that 60 guaranteed me winning because "1, 2, 3, 4, 5 and 6 are all factors of 60. There are no other numbers under 100 that have all these numbers as factors". I pressed Max on his thinking – what did it mean for these numbers to be a factor of 60? Max replied that it meant we could divide 60 by any of these numbers and get a whole number answer. Max had made significant progress with the extending prompt. In particular, he had identified 720 as a "very good number" because it had the numbers 1 to 6 (and 10) as factors, and also 8 and 9. He had identified this number using the sophisticated strategy of setting out all the multiples of 60 (60, 120, 180 etc), and using what he knew about multiplication patterns to identify which of these numbers were potentially multiples of 7, 8 and 9. However, because Max's thinking was significantly more sophisticated than most of his classmates, it was decided to end the discussion without considering the extending prompt.

Consolidating and extending the learning

Recently, advocates of teaching through problem-solving have emphasised the importance of providing students with follow-up experiences to consolidate their understanding and promote generalising (Sullivan et al., 2015). These experiences may take many forms, including a follow-up challenging task that is of similar level of cognitive demand (Sullivan et al., 2015), students undertaking more routine tasks (Hopkins & Russo, 2017), or a purposeful mathematical game (as outlined here). Sometimes these consolidating experiences will occur in the same lesson, other times they are planned for subsequent lessons. To follow-up the Lucky Dice investigation, I recommended that the classroom teacher build a second session around the game Skip-Counting Bingo (see textbox), which explores the same concept through a similar context.

Consolidating activity: Skip-counting bingo (Lesson 2)

Materials: 100-chart, Dice (6-sided, 10-sided, 12-sided, or 20-sided dice, depending on desired level of challenge)

This game is for 2 to 5 players.

To begin, children each choose five Bingo numbers in turn, and mark these numbers on their 100-chart. Note that players must choose numbers greater than 10 (or 20).

One of the children rolls the dice. Together, children begin counting by whatever the number rolled, using the 100-chart to keep track. For example, if they roll a four, they would begin counting by 4's from zero: 4, 8, 12, 16, 20 etc. Children stop counting when they encounter a bingo number. In the game shown in Figure 2, children would stop counting at 28 if a four was rolled, as this is the first Bingo number encountered. This number is removed from the board.

The dice is rolled again, and a new counting sequence is explored. For example, if a 3 is rolled, the group would be counting by 3's from zero. They would again stop when they encountered a bingo number (42 in Figure 3). Play continues until one of the players removes all their numbers and shouts 'Bingo!'.

Having undertaken this pair of lessons, a reasonable question might be: "Where should students go next in their learning?"

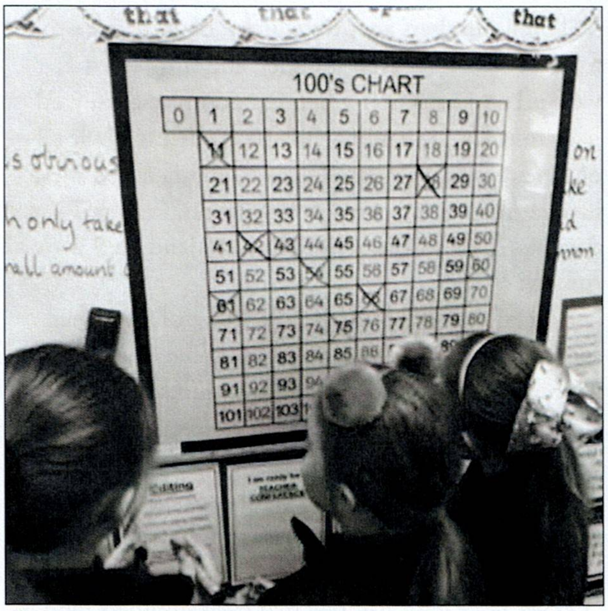

Figure 3. Three children begin a game of Skip-Counting Bingo.

Many note that when learning through problem-solving based approaches, presenting and working through isolated tasks can be problematic (Anghileri, 2006), and that such tasks need to be purposefully sequenced to best support student learning (Sullivan et al., in press).

One design principle for supporting the sequencing of tasks is variation theory. The premise of variation theory is that deliberately varying only one aspect of a task can effectively support learning, as it focuses student attention on what is different between the two versions of a task (Kullberg, Kempe, & Marton, 2017). One means of interpreting this idea is to keep the learning context the same, while varying the concept. The argument is that this approach can deepen understanding and help build connections between different mathematical domains (Sullivan et al., in press).

In our case, we may wish to introduce our Year 4 students to the notion of non-linear counting patterns, specifically repeated doubles patterns, through designing a follow-up task labelled Lucky Dice Again (see textbox). This extending task is identical in its presentation to the original Lucky Dice task, except the mathematical process is now repeatedly doubling numbers rather than skip-counting. Students exploring the Lucky Dice Again task will learn that doubling patterns (i.e., multiplying successive terms in a number sequence by two) cannot be directly equated with any particular skip-counting pattern. They may also notice that repeated doubling patterns grow much faster than skip-counting patterns. These contrasting experiences are laying the foundation for students to distinguish between arithmetic and geometric sequences.

The task: Lucky Dice Again (Lesson 3)

My dad offered me another deal. I choose any number on a hundreds chart. He'd then roll a 6-sided dice, and we'd keep doubling whatever number he rolled. If we land on my number, he'd give me 10 dollars. If we skip my number, I'd give him 10 dollars. What are some good numbers I could choose? Should I take the deal?

Finally, having explored the Lucky Dice Again task, there is a need to consolidate students understanding of repeated doubling patterns. This consolidation might occur through another game-based activity, designed as a carefully constructed variant on Skip-Counting Bingo; Doubles Bingo (see textbox). Again, the mechanics of the game are identical, but for the fact that, rather than skip-counting, students are repeatedly doubling numbers.

Consolidating activity: Doubles Bingo (Lesson 4)

Materials: 100-chart, Dice (6-sided, 10-sided, 12-sided, or 20-sided dice, depending on desired level of challenge)

This game is for 2 to 5 players.

To begin, children each choose five Bingo numbers in turn, and mark these numbers on their 100-chart. Note that players must choose numbers greater than 20.

One of the children rolls the dice. Together, children begin doubling whatever the number rolled. For example, if they roll a four, they would begin doubling 8, 16, 32, 64, 128. Children stop doubling when they encounter a bingo number and this number is removed from the board.

The dice is rolled again, and a new counting sequence is explored. For example, if a 3 is rolled, the group would be doubling from 3 (6, 12, 24, 48, 96, 192). They would again stop if/when they encountered a bingo number.

Play continues until one of the players removes all their numbers and shouts 'Bingo!'.

This sequence of four lessons is represented by the Venn Diagram displayed at Figure 4. The overlap in the circles is used to denote when tasks have been deliberately designed to have aspects in common. For example, the Doubles Bingo game in the fourth lesson is exploring the same concept as the Lucky Dice Again task (Lesson 3), in the same context as the Skip-Counting Bingo task (Lesson 2).

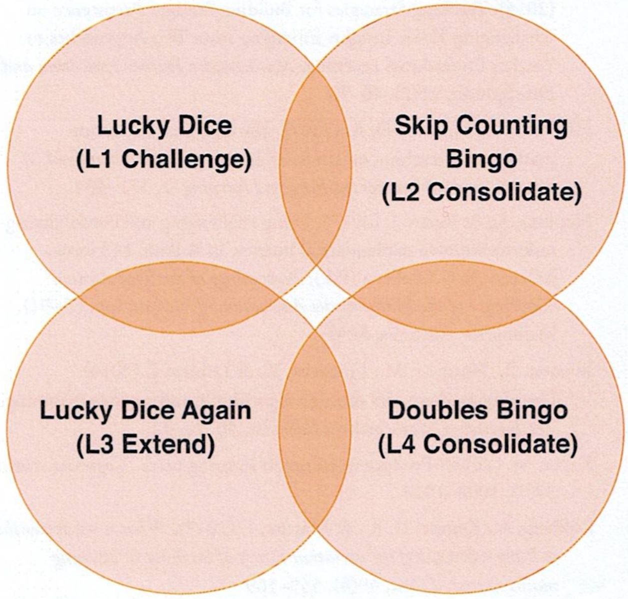

Figure 4. Sequence of four lessons adopting a Challenge-Consolidate-Extend-Consolidate structure.

Concluding remarks

Teaching with challenging tasks has enormous potential to support student learning in mathematics (Sullivan et al., in press). In this article, I have described my experience teaching with one of my favourite challenging tasks (Lucky Dice), which, in my experience, can be used effectively with students as young as Year 2 right through to Year 6. In the second part of the article, I have spent some time outlining how we might build a mini-sequence of learning from the Lucky Dice task through drawing on principles of variation theory. This mini-sequence is summarised in Figure 4, which uses the tasks discussed in this article as an example of a Challenge-Consolidate-Extend-Consolidate lesson sequence structure. Hopefully primary teachers find this framing to be of some value when considering how to plan for multiple connected learning experiences based around a core challenging task.

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