Teachers' Guide: Assessing Reasoning

This Special Topic (ST5) Assessing Reasoning provides the Assessing Mathematical Reasoning Rubric for assessing reasoning together with illustrations of how teachers have used this resource in their classrooms.

The Teachers' Guide provides:

- The aims and rationale for this special topic.
- An explanation of mathematical reasoning.
- Definitions of the main reasoning actions and learning trajectories for these actions.
- A process and Assessing Mathematical Reasoning Rubric for assessing students' mathematical reasoning.
- An introduction to the exemplar materials – the tasks used to gather examples of students' reasoning and teachers' assessment of their reasoning.

Teachers can use this resource to find out about their students' reasoning proficiency and take action to improve this proficiency for the whole class and for individual students. This might result in teachers using specific reasoning tasks in their lessons, or taking the opportunity to elicit and challenge students to reason in their lessons.

Aims

The **Assessing Reasoning** Special Topic (ST5) aims to assist teachers to encourage and notice reasoning and to conduct **formative** assessment of reasoning, one of the four proficiencies in the Australian Curriculum: Mathematics. It also aims to assist teachers to provide opportunities for students to reason in all mathematics lessons.

Rationale

Reasoning is one of four proficiencies in the Australian Curriculum: Mathematics. Previous research involving Australian teachers shows that teachers struggle to include reasoning in their lessons. They often associate reasoning with explaining and many do not make reference to the other reasoning actions listed in the Australian Curriculum. Some teachers also find it difficult to distinguish between reasoning and problem solving.

Reasoning is active. It is the mathematical thinking that creates and validates mathematical ideas and new knowledge, and it is meaning making. Students discover and make sense of mathematical ideas and concepts when engaged in inquiry-based tasks that require them to reason. Reasoning tasks in mathematics provides opportunities for creative thinking. Hence reasoning is intertwined with the other proficiencies: understanding, fluency and problem-solving. These resources show these connections between reasoning and the other proficiencies and the way in which understanding develops through reasoning.

When researching and trialling the resources provided here teachers have reported that conducting lessons that include reasoning and using the formative assessment rubric has enabled them to realise the importance of language in mathematics and for students to be able to communicate their ideas and thinking. They have also noticed that some students surprise them and either do better than expected or not as well as expected on these tasks. This enabled the teachers to re-evaluate the learning needs of their students and consider the different levels of engagement of their students.

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Reasoning involves a number of different actions. These are listed in the Australian Curriculum: Mathematics but not clearly defined in the curriculum statements; nor are there clear reasoning learning outcome statements which show how capability with these reasoning actions develops to show improved proficiency. In these resources reasoning actions are defined, elaborated, and illustrated. The Assessing Mathematical Reasoning Rubric included in this resource is designed to assist teachers to assess students' reasoning irrespective of the mathematical content of the task. Formative assessment is most appropriate since there are no outcome statements for year levels included in the Australian Curriculum and students at various year levels are able to reason with high levels of proficiency for appropriate content. Further explanation of the Assessing Mathematical Reasoning Rubric and the process of formative assessment that teachers can follow when using this resource is provided in the section Assessing Reasoning.

About Reasoning

Reasoning in the Australian Curriculum: Mathematics is defined as:

> Students develop an increasingly sophisticated capacity for logical thought and actions, such as **analysing**, **proving**, **evaluating**, **explaining**, **inferring, justifying** and **generalising**. Students are reasoning mathematically when they **explain** their thinking, when they **deduce** and **justify** strategies used and conclusions reached, when they **adapt** the known to the unknown, when they **transfer** learning from one context to another, when they **prove** that something is true or false and when they **compare** and **contrast** related ideas and **explain** their choices.

The key idea here is logical thought. This involves being convincing and providing evidence, just as is required in argumentative writing. Explaining a process for doing something is not convincing when you have asked the question 'why?' in a conversation. It is the same in mathematics, explaining steps in a process for solving a problem is not convincing because it does not answer the question 'why?'

There are many reasoning actions listed in the Australian Curriculum definition which all contribute to logical thought and creating new knowledge and understanding for the learner. These differ from the verbs included in the problem solving statement from the Australian Curriculum as shown in the following Table 1. For problem solving students need to make sense of the problem by interpreting it and then make choices about how to represent it or model it mathematically and decide how to solve it. Communicating effectively involves explaining the process for solving the problem as well as showing that the solution makes sense for the problem. The reasoning verbs are quite different, for example generalising, justify, proving and inferring. Analysing, explaining and evaluating might seem relevant to problem solving but they have a different purpose when reasoning. Analysing when problem solving involves interpreting the problem; analysing when reasoning is about comparing and contrasting to notice similarities and differences and patterns. The explaining used in problem solving is 'explaining how', whereas the types of explaining used when reasoning are 'explaining what' and 'explaining why.'

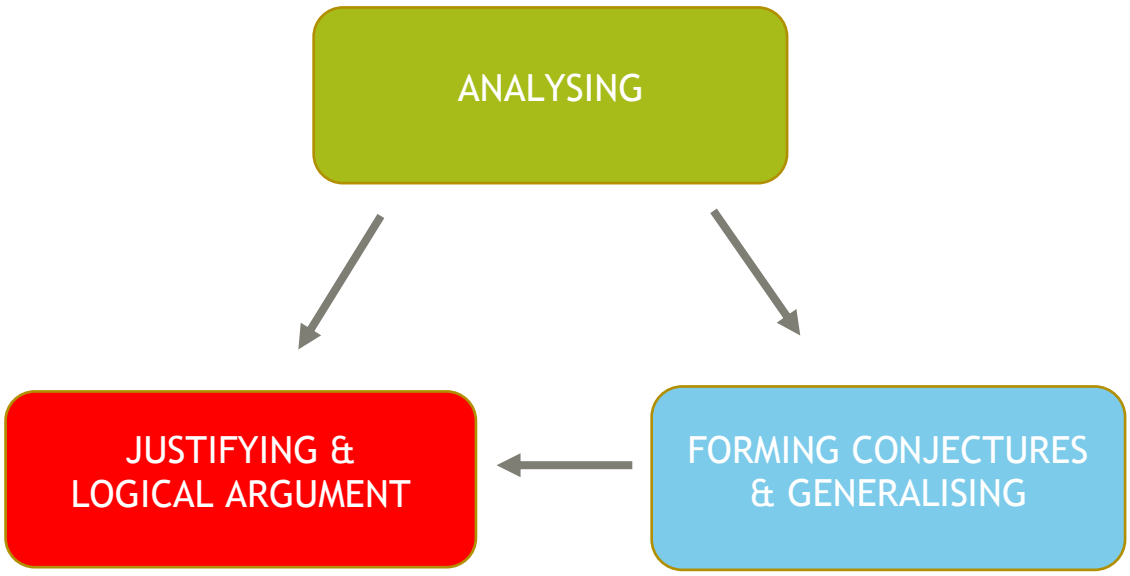
Table 1. Actions and capabilities for the reasoning and problem solving proficiencies in the Australian Curriculum (ACARA, 2017)

Reasoning	Problem Solving
Students develop capacity for logical thought and actions such as: analysing proving evaluating explaining inferring justifying generalising	Students develop the ability to: make choices interpret formulate, model and investigate problem situations communicate solutions effectively.

The reasoning actions listed in the Australian Curriculum are related to each other. For example, proving and evaluating are both forms of justifying, and inferring is required to generalise.

These Assessing Reasoning resources focus on three main reasoning actions: Analysing, Forming Conjectures and Generalising, and Justifying and Logical argument. These three reasoning actions incorporate the other verbs describing reasoning that are included in the Australian Curriculum: Mathematics.

Some mathematics tasks will require students to use all three reasoning actions, such as the 'What Else Belongs?' task included in this resource. Other tasks may provide students with a statement that is thought to be true but not known or shown to be true, that is a conjecture or generalisation, and students then are required to test the conjecture or justify the statement and so will use only analysing, and justifying and logical argument, for example the task 'Is it Right?' The connections between these three reasoning actions are shown in the following figure. It shows that you normally start a reasoning task by analysing.

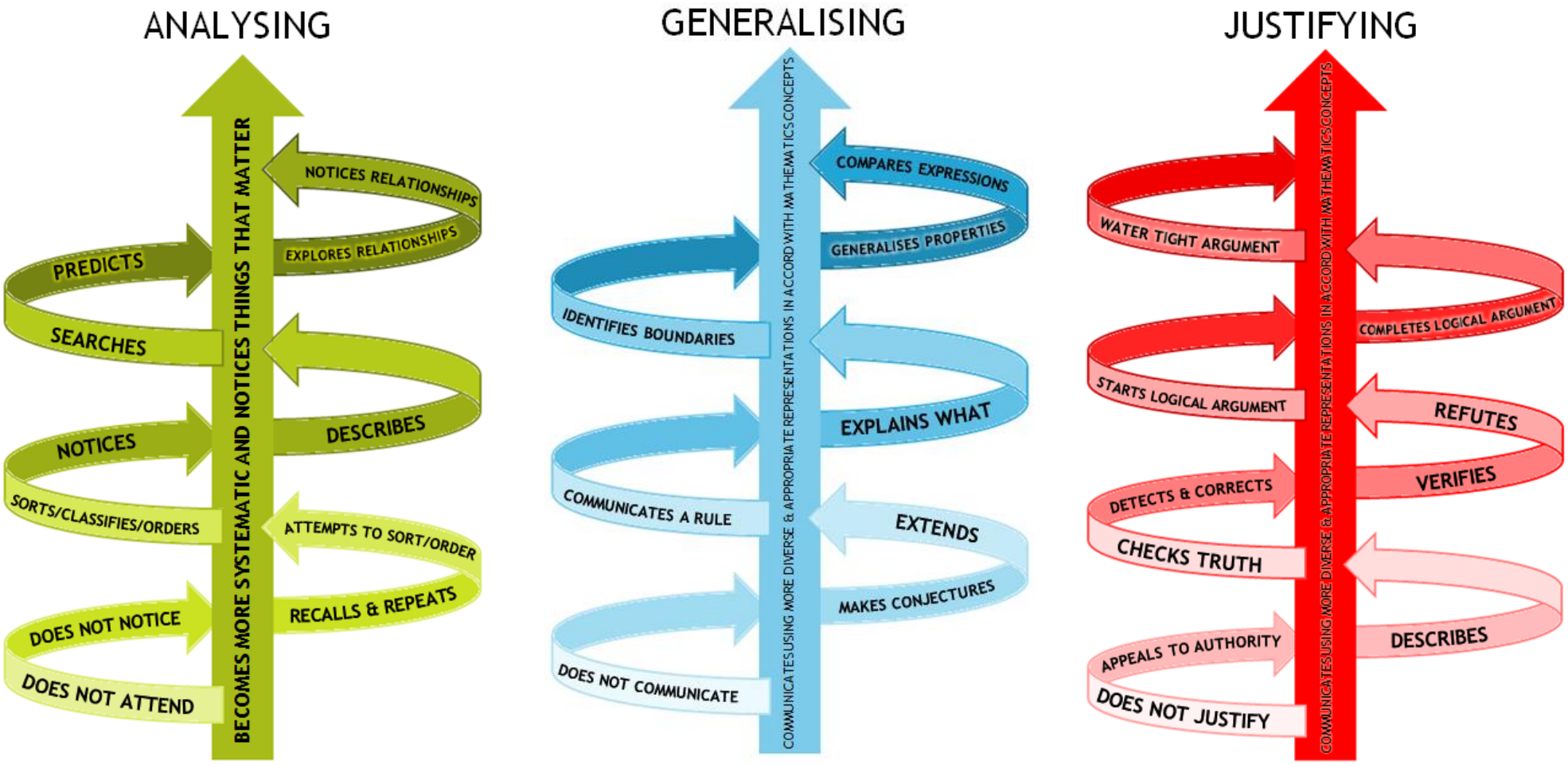


These three actions are defined in the following table. You will notice that there is more than one way in which each reasoning action might occur or more than one element that defines this action. This is because different reasoning tasks will elicit particular types of analysing, forming conjectures and generalising, or justifying and logical argument. The multiple elements also indicate that the reasoning action can be performed at different levels of proficiency.

Reasoning actions: Definitions

Analysing	Forming Conjectures and Generalising	Justifying and Logical Argument
Analysing involves ***exploring*** the problem using examples provided or generating examples to form or test a conjecture. Analysing occurs by ***comparing and contrasting*** cases to notice: • what is same and what is different, and to sort and classify the cases. • what stays the same and what changes and to recall, repeat or extend the pattern. Analysing involves using ***numerical or spatial structure***, ***known facts or properties*** when sorting cases or repeating and extending pattern. Categories of cases and patterns are identified by *labelling* using ***terms, diagrams or symbols***.	Forming conjectures involves ***developing statements*** that are thought to be true but not yet known or shown to be true. Generalising involves ***identifying common*** *properties* or patterns across more than one case and ***communicating a rule (conjecture)*** to describe the common property, pattern or relationship. The ***statement or rule is communicated*** orally, written, diagrams or symbols. The ***meaning of the statement or rule is communicated*** using ***particular examples*** to explain the property or pattern. Further examples are used to explain how the rule applies to other cases.	Justifying involves ***checking the truth of conjectures and generalisations*** to demonstrate or refute the truth of a claim. Justifying uses ***logical argument*** to convince others of the truth of the claim or to refute the claim. A ***logical argument*** is made by: • using ideas that are already understood; • following agreed processes or steps for making arguments; and • using terms, diagrams and symbols that are known and understood. A ***mathematical refutation*** involves demonstrating that a particular statement is false. ***Types of logical argument include***: • verifying and validating the claim by showing how the rule works for all cases (exhaustion), • refuting a claim by providing a counter example, • refuting a claim by providing a contradiction, that is showing that two statements are not equivalent, related or connected (contradiction), • deduction that is using one or more logical steps – if… then … • uses general representation of the claim along with logical steps to show how the claim applies to all cases (generic proof) **A logical argument includes <u>every single step in the reasoning process</u>, leaving nothing unexplained.**

Reasoning Learning Trajectories



These three trajectories indicate students' progression as they develop their capacity with these three main reasoning actions.

As students develop their capacity to **analyse** they become more systematic and more often notice things that matter. The steps in this progression are summarised as: do not notice (common properties, patterns or relationships), recall and repeat, attempt to sort and order; sorts, classifies and orders, notices, describes; searches, predicts, explores relationships, notices relationships.

As students develop their capacity to **form conjectures and generalise** they communicate their conjectures and generalisations using more diverse and appropriate representations in accord with mathematics concepts. The steps in this progression are summarised as: does not communicate a conjecture or generalisation, makes a conjecture, extends (the conjecture to include more cases), communicates a rule, explains what (the rule means); identifies boundaries (for the rule); generalises properties and compares expressions (or rules and generalisations).

As students develop their capacity to **justify and form logical arguments** they communicate their arguments using more diverse and appropriate representations in accord with mathematics concepts. The steps in progressing their capacity to justify and form logical arguments are: does not justify or appeals to authority (the teacher or other source), describes; checks the truth, detects and corrects (incorrect or untrue conjectures/generalisations); verifies or refutes conjectures/generalisations, starts a logical argument; completes the argument; or finally provides a watertight argument.

The steps in these reasoning learning trajectories are described more fully in the Assessing Mathematical Reasoning Rubric. You might notice that as you progress along each trajectory the colour changes. These match with the different levels of reasoning included in the Rubric.

Assessing Reasoning

Students communicate their reasoning through gesture, talking and writing using diagrams, words, and symbols. To notice and assess reasoning teachers need to watch and listen to their students as well as review what they write and record.

These resources are designed to be used as formative assessment or assessment for learning.

> Assessment ***for*** learning involves teachers using evidence about students' knowledge, understanding, and skills to inform their teaching. Sometimes referred to as 'formative assessment', it usually occurs throughout the teaching and learning process to clarify student learning and understanding. (http://syllabus.nesa.nsw.edu.au/support-materials/assessment-for-as-and-of-learning/)

It differs from summative assessment or assessment of learning which is used to measure student achievement to determine what standard they have achieved:

> Assessment ***of*** learning assists teachers in using evidence of student learning to assess achievement against outcomes and standards. Sometimes referred to as 'summative assessment', it usually occurs at defined key points during a unit of work or at the end of a unit, term or semester, and may be used to rank or grade students. The effectiveness of assessment of learning for grading or ranking depends on the validity and reliability of activities. Its effectiveness as an opportunity for learning depends on the nature and quality of the feedback. (http://syllabus.nesa.nsw.edu.au/support-materials/assessment-for-as-and-of-learning/)

When conducting formative assessment of reasoning teachers are aiming to find out how students analyse, generalise and justify and how they communicate their findings and argument. The teachers may need to use prompts to elicit students' reasoning. This may be done when interacting with students or when setting requirements for recording their responses to the task. Teachers' assessment of children's reasoning in the moment may lead them to use supporting and enabling prompts to get students started, or use challenging prompts to push them to explore more deeply and reason more convincingly. Teachers may also collect students' written or recorded work. This might include recordings using digital tools that enable students to write, draw and verbally communicate their reasoning as a couple of teachers did when trialling these resources. Teachers' can then take the time to assess the reasoning of all students in the class. This provides the teacher with information about the reasoning proficiency of the students and the diversity in the class that the teacher can then use for planning their lessons to enhance students' reasoning. This planning may involve selecting particular reasoning tasks that provide

more opportunities for reasoning or to more consciously use prompts to elicit, support and challenge students' reasoning in all lessons.

Rubric for assessing mathematical reasoning

The Assessing Mathematical Reasoning Rubric (on the next page) is designed for teachers to identify both the type of reasoning action that the student is using and the level of proficiency that the student is demonstrating.

The Assessing Mathematical Reasoning Rubric is an elaboration of the reasoning trajectories to describe the definitions of the three main reasoning actions as levels of proficiency that can be developed when students are offered support and opportunities to reason. Reasoning proficiency for primary students (the steps in the reasoning trajectories) are organised into five (5) levels: not evident, beginning, developing, consolidating, and extending. These levels do NOT relate to particular year levels, though if reasoning is regularly included in mathematics lessons we would expect students in the upper primary grades to be performing in the upper levels. Even so, quite young students are able to reason at the higher levels with tasks where the content is appropriate for their level of understanding, for example, providing a watertight argument.

There are a number of descriptors at the levels for each reasoning actions. These describe the types of reasoning that may occur at this level. They also take into account the different types of tasks that involve reasoning. For example, with respect to analysing, some tasks may require noticing common properties to form a conjecture, such as "What else belongs?" whereas other tasks may require recalling facts, such as 'Is it right?' to form an argument.

Each exemplar provides examples of students' reasoning that match the elements and level of reasoning in this framework. In addition to matching student responses with elements within this Assessing Mathematical Reasoning Rubric, the table provided in the exemplar also identifies supporting and challenging prompts that the teacher could use to support the student to demonstrate a higher level of reasoning.

	Analysing	Forming Conjectures and Generalising	Justifying and Logical argument
Not evident	• **Does not notice** numerical or spatial structure of examples or cases. • Attends to **non-mathematical** aspects of the examples or cases.	• **Does not communicate** a common property or rule for pattern. • **Non-systematic** recording of cases or pattern. • **Random** facts about cases, relationships or patterns.	• **Does not justify**. • **Appeals** to teacher or others.

Beginning	• **Notices** similarities across examples
	• **Recalls** random known facts related to the examples.
	• Recalls and **repeats patterns** displayed visually or through use of materials.
	• Attempts to **sort** cases based on a common property.
		• Uses body language, drawing, counting and oral language to **draw attention** to and **communicate**:
		  ○ a single common property
		  ○ repeated components in patterns.
		• **Adds** to patterns displayed verbally and/or visually using diagrams or through use of materials.
			• **Describes** what they did and why it may or may not be correct.
			• **Recognises** what is correct or incorrect using materials, objects, or words.
			• Makes **judgements** based on simple criteria such as known facts.
			• The **argument may not be coherent** or include all steps in the reasoning process.

Developing	• **Notices** a common numerical or spatial property.
	• **Recalls**, **repeats and extends** patterns using numerical structure or spatial structure.
	• **Sorts and classifies** cases according to a common property.
	• **Orders** cases to show what is the same or stays the same and what is different or changes.
	• **Describes** the case or pattern by labelling the category or sequence.
		• **Communicates** a rule about a:
		  ○ *property* using words, diagrams or number sentences.
		  ○ *pattern* using words, diagrams to show recursion or number sentences to communicate the pattern as repeated addition.
		• **Explains** the meaning of the rule using one example.
			• **Verifies** truth of statements by using a common property, rule or known facts that confirms each case. May also use materials and informal methods.
			• **Refutes** a claim by using a counter example.
			• Starting statements in a **logical argument** are correct and accepted by the classroom.
			• **Detecting and correcting errors** and inconsistencies using materials, diagrams and informal written methods.

Consolidating	• **Notices more than one** common property by systematically generating further cases and/or listing and considering a range of known facts or properties.
	• **Repeats and extends patterns** using both the numerical and spatial structure.
	• Makes a **prediction** about other cases:
	  ○ with the same property
	  ○ included in the pattern.
		• **Identifies** the boundary or limits for the rule (generalisation) about a common property.
		• **Explains** the rule for finding one term in the pattern using a number sentence
		• **Extends** the number of cases or pattern using another example to explain how the rule works.
			• Uses a correct **logical argument** that has a complete chain of reasoning to it and uses words such as 'because', 'if...then...', 'therefore', 'and so', 'that leads to' ...
			• **Extends the generalisation** using logical argument.

Extending	• **Notices and explores** relationships between:
	  ○ common properties
	  ○ numerical structures of patterns.
	• **Generates examples**:
	  ○ using tools, technology and modelling
	  ○ to form a conjecture.
		• **Communicate**s the rule for any case using words or symbols, including algebraic symbols.
		• **Applies** the rule to find further examples or cases.
		• **Generalises** properties by forming a statement about the relationship between common properties.
		• **Compares** different symbolic expressions used to define the same pattern.
			• Uses a **watertight logical argument** that is mathematically sound and leaves nothing unexplained.
			• **Verifies** that the statement is true or the generalisation holds for *all* cases using logical argument.

Evidence of student's reasoning (work sample and orally).

Using the Assessing Mathematical Reasoning Rubric

The exemplars included in this resource show how teachers used the Assessing Mathematical Reasoning Rubric to assess students' reasoning. The annotated work samples included in each exemplar provide the evidence that teachers used when assessing a students' written work.

Some teachers were able to use the Assessing Mathematical Reasoning Rubric in the classroom while interacting with students to assess their reasoning. They highlighted actions in the Assessing Mathematical Reasoning Rubric using a highlighter and then recorded written descriptions of the evidence observed during the class or in the students' written work samples (see exemplars).

Other teachers used the Assessing Mathematical Reasoning Rubric as they analysed each piece of written work, collaborating with a peer to support each other in this task. This collaboration provided moderation of their assessment, where they reached agreement on the reasoning actions observed. Keep in mind that observing and listening to students may provide other evidence that would identify different reasoning actions and even levels of reasoning.

When using the Assessing Mathematical Reasoning Rubric it is important to realise that students may not be at the same level for each of the three reasoning actions for a given task. For example they may be considered 'consolidating' for analysing, 'developing' for generalising, and 'beginning' for justifying (or some other combination).

The level of students' reasoning actions may vary from task to task depending on content. This is because reasoning is intertwined with the development of other proficiencies, especially understanding, so that students' level of reasoning may be influenced by their understanding or fluency. For example, students with a strong number sense may be able to construct a logical argument about number, but if they are unfamiliar with geometry properties may demonstrate lower levels of justifying and logical argument when working on a geometry reasoning task.

Topic Outline

General outline

- The following exemplars are intended to provide a sense of what mathematical reasoning learning experiences look like from Years 3-6 and the way teachers can formatively assess the reasoning actions of their students.
- The exemplars offer ways to *plan* tasks that embed the reasoning actions into everyday lessons: ways to *notice* mathematical reasoning when it happens and ways to *assess* student reasoning.
- Each exemplar contains a table of sample students' responses to the task that aligns each response to a reasoning action, element in the Assessing Mathematical Reasoning Rubric, level for that reasoning action and suggested reasoning prompts.
- Each exemplar contains *suggested reasoning prompts* that can support teachers in their questioning to support and/or extend students' reasoning. A full list of reasoning prompts, together with prompt cards can be found in this Teacher's Guide.
- Each exemplar contains a student *annotated work sample* that identifies the evidence that the teacher used to determine the reasoning actions demonstrated and level for each reasoning action.
- Each exemplar contains a teacher's *formative assessment of a student's reasoning* using the Assessing Mathematical Reasoning Rubric, that is highlighting the elements from the rubric that were demonstrated and a written comment regarding evidence.
- The exemplars are not connected to one another. They are intended to be used as individual tasks and included in relevant sequences of lessons.
- Each exemplar is designed to run for one lesson but may be extended to run over more than one lesson depending on the class or time available.

Task selection

Exemplar	**Recommended Year Level** (also suited to other year levels)	Brief Description of Activities or Goals etc.	Australian Curriculum Links (optional)
Is it Right?	**3** (4, 5)	**Number focus:** • Place Value • Additive Thinking **Reasoning focus:** • Exploring and noticing relationships between numerical structures (analysing). • Verifying or refuting statements as true (justifying)	**ACMNA053:** Apply place value to partition, rearrange and regroup numbers to at least 10 000 to assist calculations and solve problems • justifying choices about partitioning and regrouping numbers in terms of their usefulness for particular calculations
Number Towers	**3** (4, 5, 6)	**Number focus:** • Additive Thinking **Reasoning focus:** • Exploring and noticing relationships between numerical structures (analysing) • Using trials to develop conjectures (analysing) • Comparing and contrasting to form conjectures • Explaining the conjecture using an example • Testing conjecture using examples to verify (justifying) • Using understanding of properties numbers to justify conjecture	**ACMNA054:** Recognise and explain the connection between addition and subtraction **ACMNA055:** Recall addition facts for single-digit numbers and related subtraction facts to develop increasingly efficient mental strategies for computation
Magic V	**4** (3, 5)	**Number focus:** • Properties of Number **Reasoning focus:** Developing and testing conjectures. This includes: • Comparing and contrasting to notice a common property • Forming a conjecture about the common property (generalising) • Explaining the conjecture using an example • Testing conjectures using trials • Using understanding of equivalence and properties of odd and even numbers to form a logical argument to justify	**ACMNA071:** Investigate and use the properties of odd and even numbers

Matchsticks	**4** (3, 5, 6)	**Number focus:** • Algebraic Thinking **Reasoning focus:** • Noticing pattern in number • Forming and testing conjectures. • Communicating a conjecture about the pattern using words or symbols (number sentences) • Using a rule to communicate the general case (generalising) • Explaining the conjecture or rule using an example • Justifying the rule	**ACMNA081:** Explore and describe number patterns resulting from performing multiplication • identifying examples of number patterns in everyday life
What Else Belongs?	**5** (3, 4, 6)	**Number focus:** • Multiplicative Thinking • Number sense **Reasoning focus:** • Notice common properties of numbers • Write a statement (conjecture) about the common property • Explain the common property • Uses the common property to find other cases • Justify their conjecture by verifying that each number belongs	**ACMNA098:** Identify and describe factors and multiples of whole numbers and use them to solve problems • exploring factors and multiples using number sequences • using simple divisibility tests
Area and Perimeter	**5, 6**	**Mathematical focus:** • Measurement • Multiplicative Thinking **Reasoning focus:** Testing conjectures. This includes: • Using trials to refute the statement • Using understanding of perimeter and area of rectangles to provide a logical argument	**ACMMG137:** Solve problems involving the comparison of lengths and areas using appropriate units • recognising and investigating familiar objects using concrete materials and digital technologies
Painted Cube	**7** (5, 6)	**Number focus:** • Algebraic Thinking **Reasoning focus:** • Forming and testing conjectures. • Generalising – writing statements in words or symbols to describe patterns • Generalising - communicates the rule for any case using words or symbols, including algebraic symbols.	**ACMNA176:** Create algebraic expressions and evaluate them by substituting a given value for each variable • using authentic formulas to perform substitutions

General Advice

This reSolve resource differs from others as it provides a tool and process for assessing mathematical reasoning together with examples of tasks that have used to gather evidence of students' reasoning proficiency. The focus of these materials is to show how the Assessing Mathematical Reasoning Rubric can be used to assess students' reasoning when responding to this task. In keeping with the primary aim is to conduct formative assessment – assessment for learning, the exemplars provided in this set of resources also include prompts that teachers may use to support and challenge students' to reason at a higher level of proficiency. Below you will find a list of prompts for each key reasoning action that teachers found useful as well as a set of prompt cards that teachers distributed to students.

Productive Group Work

While many of the tasks can be completed in pairs or in small groups, we encourage students to 'have a go' on their own first and then gravitate towards a partner/small group to discuss/compare/add to their thinking and reasoning.

Prompts we find useful for encouraging students to listen to one another include:

- "Can you explain what [student] said?"
- "Who had a similar/different idea?"
- "Be prepared to share your thinking."

Further prompts for each reasoning action are listed below.

Notes from Trialling

1. Something to consider is the limitation of assessing reasoning solely based on a student work sample. In many of our lessons, teachers reported the value in audio or video recording, often with iPads, as students shared their thinking aloud and this completed the work sample and supported teachers' decisions when assessing against the Assessing Mathematical Reasoning Rubric. It is important to remember that reasoning is *active* and usually involves students communicating their reasoning orally.
2. When assessing using the Assessing Mathematical Reasoning Rubric, teachers reported the 'Evidence of student's reasoning" helpful in making comments about both the work sample and students orally communicated to support the teachers' assessment decisions.
3. While having physical reasoning prompt cards is optional, teachers reported that having the laminated class sets for students was helpful in focusing students' attention to the reasoning aspect of the task and supporting and/or extending their reasoning.
4. When in doubt of what prompt to use, simply ask the student "Why?"

Prompts to Elicit Mathematical Reasoning

The following prompts may assist you to elicit mathematical reasoning within your classroom. Reasoning Prompt Cards to print and use in the classroom can be found on the following pages.

Analysing Prompts

- What is the same and different about …?
- What stays the same and what changes?
- Sort or organise the following according to …
- Alter an aspect of something to see (such and such) an effect. If we change this what will happen? What follows from this?
- What do you think will happen next if we did this?

Forming Conjectures and Generalising Prompts

- What is the pattern here?
- Is that … (pattern) always going to work?
- What happens in general?
- What is the rule?
- Are there other examples that fit the rule?
- How can you explain the rule to someone else?

Truth Prompts

- Is (this proposition) true?
- Is it just sometimes true, or is it always true?
- When is it true?
- How do you know?
- How could we demonstrate/show/prove that it is true?
- True or false? Why? Let's justify.
- Convince me.

Justifying and Logical Argument Prompts

- Convince me …
- How can we be sure …? How do you know…?
- Tell me what is wrong with …
- Explain - why does this (process / procedure / result) work?
- Can you go through that step by step?
- Why?

References:

Victorian Curriculum and Assessment Authority. (2002). CSF II Reasoning and Strategies Levels 1-6. (p. 27).
The Australian Association of Mathematics Teachers (AAMT) Inc. (2016). Top Drawer.
http://topdrawer.aamt.edu.au/Reasoning

Reasoning Prompt Cards

Analysing Prompt Cards

What is the same and different about …?	What stays the same and what changes?
Sort or organise the following according to …	Alter an aspect of something to see (such and such) an effect. If we change this what will happen?
What follows from this?	What do you notice…?
"If…then…."	What do you think will happen next if we did this?

What is the pattern here?	Is that … (pattern) always going to work?
What happens in general?	What is the rule?
Are there other examples that fit the rule?	How can you explain the rule to someone else?

Is this true? How do you know?	Is it just sometimes true, or always true?
When is it true?	How do you know?
How could we demonstrate/ show/prove that it is true?	True or false? Why? Let's justify.

Convince me…	How can we be sure…?
How do you know….?	Tell me what is wrong with….
Explain why this works/doesn't work.	Can you go through that step by step?

Further Information

References

Australian Curriculum Assessment and Reporting Authority. (2015). Australian curriculum: Mathematics. Sydney: Author.

Victorian Curriculum and Assessment Authority. (2002). CSF II Reasoning and Strategies Levels 1-6. (p. 27).

The Australian Association of Mathematics Teachers (AAMT) Inc. (2016). Top Drawer. http://topdrawer.aamt.edu.au/Reasoning

Further reading

Magic V

- Bragg, L. A., Loong, W., Widjaja, W., Vale, C., & Herbert, S. (2015). Promoting reasoning through the Magic V task. *Australian Primary Mathematics Classroom*, *20*(2), 10-14. Retrieved from www.aamt.edu.au/content/download/34315/490073/file/apmc20_2_bragg.pdf

What Else Belongs?

- Bragg, L. A., Vale, C., Herbert, S., Loong, E., Widjaja, W., Williams, G. and Mousley, J. (2013). Promoting awareness of reasoning in the primary mathematics classroom, In A. McDonough, A. Downton, and L. A. Bragg, (Eds.), *Mathematics of the planet earth : Proceedings of the MAV 50th Annual Conference 2013*, pp. 23-30, Mathematical Association of Victoria, Melbourne, Vic.
- Vale, C., Bragg, L. A., Widjaja, W., Herbert, S., & Loong, E.Y. (2017). Children's mathematical reasoning: Opportunities for developing understanding and creative thinking. *Australian Primary Mathematics Classroom*, 22*(1)*, 3-8.

Bragg, L. A., Herbert, E. S., Loong, E. Y. K., Vale, C., & Widjaja, W. (2016). Primary teachers notice the impact of language on children's mathematical reasoning. *Mathematics education research journal*, *28*(4), 523-544. DOI:10.1007/s13394-016-0178-y.

Bragg, L. A., Widjaja, W., Loong, E., Vale, C. & Herbert, S. (2015). Promoting reasoning through the Magic V task, *Australian Primary Mathematics Classroom, 20*(2), 10-14.

Carpenter, T. P., Franke, M., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary school*. Portsmouth: Heineman.

Clarke, D. M., Clarke, D. J., & Sullivan, P. (2012). Reasoning in the Australian Curriculum: Understanding its meaning and using the relevant language. *Australian Primary Mathematics Classroom*, *17*(3), 28-32.

Herbert, S., Vale, C., Bragg, L. A., Loong, E., Widjaja, W. (2015). A framework for primary teachers' perceptions of mathematical reasoning. *International Journal of Educational Research*, 26-37. DOI: 10.1016/j.ijer.2015.09.005.

Herbert, S., Widjaja, W., Bragg, L. A., Loong, E., & Vale, C. (2016). Professional learning in mathematical reasoning: Reflections of a primary teacher. In B. White, S. Trenholm & M. Chinnappan (Eds.), *Opening Up Mathematics Education Research Proceedings of 39th Annual Conference of MERGA,* (pp. 277-284). Adelaide: MERGA.

Hourigan, M. & Leavy, A. (2015). Geometric growing patterns: What's the rule? *Australian Primary Mathematics Classroom, 20*(4), 31-40.

Kilpatrick, J., Swafford, J., Findell, B. (Eds.) (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Lannin, J., Ellis, A. & Elliott, R. (2011). *Developing essential understanding of mathematical reasoning for teaching mathematics in prekindergarten - grade 8*. Reston, VA: National Council of Teachers of Mathematics.

Loong E. Y-K, Vale, C., Herbert, S., Bragg, L. A., Widjaja, W. (2017). Tracking change in primary teachers' understanding of mathematical reasoning through demonstration lessons. *Mathematics Teacher Education and Development, 19*(1), 5-29.

Vale, C., Widjaja, W., Herbert, S., Loong, E. & Bragg, L.A. (2017). Mapping variation in children's mathematical reasoning: The case of 'What else belongs?" *International Journal of Science and Mathematics Education*, *15*(5), 873-894. DOI 10.1007/s10763-016-9725-y

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Other recommended resources

Below is a list of suggested websites offering an abundance of free resources that promote mathematical reasoning. The resources in these sites offer lesson suggestions that teachers can use in conjunction with the Assessing Mathematical Reasoning Rubric to support their everyday planning, teaching and assessment of students' mathematical reasoning.

Organisation	Description	Link
reSolve	The reSolve project is an Australian national initiative funded by the Department of Education and Training and provides detailed materials to support teachers in facilitating mathematical inquiries in their classrooms.	http://resolve.edu.au/
Top Drawer	Australian Association of Mathematics Teachers (AAMT). Contains descriptions of the 'Big Ideas' for reasoning and reasoning-specific tasks along with other recommendations to support teachers in teaching with a reasoning focus in mind.	https://topdrawer.aamt.edu.au/Reasoning
ACARA	Australian Curriculum Assessment and Reporting Authority. This resource is a recent addition to the Australian Curriculum website that contains work samples demonstrating student reasoning.	http://resources.australiancurriculum.edu.au/proficiencies/mathematics-portfolios/reasoning/
TTML	Task Types and Mathematical Learning is a set of classroom activities designed by Victorian classroom teachers as part of a research project conducted by Monash University and the Australian Catholic University.	http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/Pages/ttml.aspx
Mathematics Task Centre	Black Douglas Professional Education Services. Contains tasks which encourage students to work like mathematicians.	http://mathematicscentre.com/taskcentre/
NRICH (UK)	NRICH: Enriching mathematics. Reasoning: Identifying Opportunities (Article)	https://nrich.maths.org/109 90
NRICH (UK)	Search for tasks by topic (content). Each lesson contains lesson descriptions for teachers, sample student solutions and additional classroom support materials.	https://nrich.maths.org/public/leg.php
NRICH (UK)	266 lessons which foster mathematical reasoning. This list can be sorted by year levels.	https://nrich.maths.org/public/leg.php?code=71
NZ Maths	Lessons and tasks according to content strands of the New Zealand curriculum	https://nzmaths.co.nz/
NCTM (USA)	National Council of Teachers of Mathematics (NCTM). Standards-based lessons and resources, including interactive tools for students.	http://illuminations.nctm.org/Lessons-Activities.aspx
NCTM (USA)	Activities with Rigor and Coherence	https://www.nctm.org/ARC s/