Interactive STEM

RESEARCH + PRACTICE BRIEF

Interactive Technologies in STEM Teaching and Learning

Productive Struggle in Mathematics

Marian Pasquale

Overarching Issues

Despite the time and effort, children will plug away for hours at a puzzle until they have solved it or at least made substantial progress. They will practice a required skill repeatedly to make the team. Why?

» What makes solving a puzzle something worth sticking with?

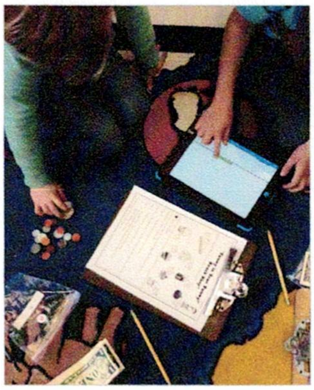
» What makes practicing a jump shot worthwhile?

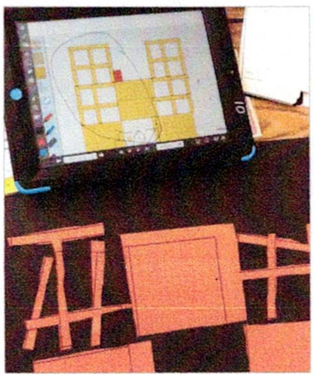
It seems that these tasks, unlike many mathematics problems, are worth the challenge because it feels as if you can make headway. "When you do believe you have a chance—when you know your own strength and the task looks accessible—challenge is appealing" (Goldenberg et al., 2015).

"Mathematics educators and researchers suggest that struggling to make sense of mathematics is a necessary component of learning mathematics with understanding" (Hiebert & Grouws, 2007). Yet, "student's struggles with learning mathematics are often viewed as a problem and cast in a negative light in mathematics classrooms" (Hiebert & Wearne, 1993; Borasi, 1996).

Unfortunately, several school factors may prevent students and teachers from viewing struggle in mathematics as a positive and productive practice. In middle and high schools, school schedules provide insufficient time and opportunity for students to grapple with challenging problems. Curriculum-pacing guides with strict calendars force teachers and students to march through topics, moving from one to the next no matter whether students have grasped the concepts. Typical norms and expectations in most mathematics classrooms tend to value behaviors that do not support good problem-solving routines. Often, individual performance is prized over group problem solving, having the right answer over reasoning and understanding, and being the first to finish over explaining and communicating.

These expectations have engendered in students the idea that mathematics insight is something one is born with. "Students who lack the skills or the confidence—ones who are





When struggle is reframed as both a natural part of the learning process and a worthwhile challenge to undertake, students are encouraged to persevere in the development of deep mathematical understanding.

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(or perceive themselves as) weak—are also typically reluctant to put in the effort that might make them stronger" (Goldenberg et. al., 2015). There is often a flawed belief that you are either good at mathematics or you are not, and struggling with a mathematical task is not viewed as an opportunity to learn but rather a weakness. The good news is that struggle can be positive and can be fostered in students with good teacher support and the right classroom norms and expectations.

What Is *Productive Struggle?*

In the Common Core Standards for Mathematical Practice, the first standard states that students should "make sense of problems and persevere in solving them" (NGA Center for Best Practices, 2010). Perseverance, or continuing forward irrespective of struggle or difficulty, is an essential element in problem solving because the first or second approach or strategy may not result in a reasonable solution. As students engage with a task, they must be mindful about the strategy they employ and assess whether it is productive. When they find they are at a dead end, they must be willing to abandon one strategy for another. When students labor and struggle but continue to try to make sense of a problem, they are engaging in *productive struggle*.

> "Great works are performed not by strength, but perseverance." — Samuel Johnson

This metacognitive ability—reflecting on one's own problem-solving approach—is developmental and takes practice and time. Productive struggle, similar to other *executive functions (i.e., cognitive skills that help the brain organize and act on information)*, is supported by a developmental progression in thinking and learning. This developmental progression can and should be nurtured. Students need support to learn how "to move through a progression or range of solution methods" (Fuson, Carroll, & Drueck, 2000). In addition, there are different kinds of struggles that students encounter as they work on a problem, for example, "when students encounter difficulty in figuring out how to get started or carry out their task, are unable to piece together and explain their emerging ideas, or express an error in problem solving" (Warshauer, 2014b). How students express these struggles is easy to spot. When beginning a task, students might say that they don't understand the directions, have never done a problem like this one before, or just stare at the paper. Students may have a "plan of attack" but are unable to implement it because they have made errors or forgotten how to proceed. Even when solving a problem correctly, explaining what they did is often their most difficult struggle.

What Factors Influence Productive Struggle?

Research suggests that students' out-of-school experiences influence their motivation and perseverance with difficult mathematical tasks. Tasks that have a familiar or real-life context are more meaningful. Because students are able to draw on their everyday experience to solve these kinds of tasks, they are more motivated to stick with the task (Taylor, 2015).

Productive struggle is also highly influenced by the socio-mathematical norms of the school class, particularly a student's perception about her mathematical ability relative to other members of the class. "Students compare themselves against the norms of mathematical competence in their class[es], construct sets of 'stories' that define their own proclivities and handicaps, and use these stories to help them decide when and to what extent they will engage in the social activity of doing mathematics" (Middleton, Tallman, Hatfield, & Davis , 2015). They often think that perseverance is something that some students are skilled at rather than a behavior that everyone can develop.

Besides students' mathematical self-image, their disposition to struggle with a challenging mathematical task depends largely on whether they (1) find the task interesting, (2) believe that they know enough mathematics to be able to solve it, and (3) believe that solving it is worth the effort (Star, 2015). A student's belief that effort is more important than innate ability is key. "In order to persevere, one needs to view the struggle that may inevitably be a part of problem solving as an opportunity to learn. Motivation enables a solver to see struggle as a natural part of the learning process, and to see that confronting and working through struggle can ultimately be helpful" (Star, 2015).

What Can Teachers Do?

The National Council of Teachers of Mathematics (NCTM) policy document, *Principles to Actions: Ensuring Mathematical Success for All*, "notes that an effective teacher provides students with appropriate challenges, encourages perseverance in solving problems, and supports productive struggle in learning mathematics" (NCTM, 2014, p.11, as cited in Star, 2015). This requires that the demand of the mathematical task is high to the extent that it provides a cognitive stretch for the student and builds on student thinking. Student struggle must be supported so that it is a positive endeavor and not one full of difficulties and frustration (Warshauer, 2014a). The kind of questions teachers ask and the kind of support that teachers offer are critical. "The kind of guidance and structure that teachers provide may either facilitate or undermine the productive efforts of students' struggles" (as cited in Warshauer, 2014a).

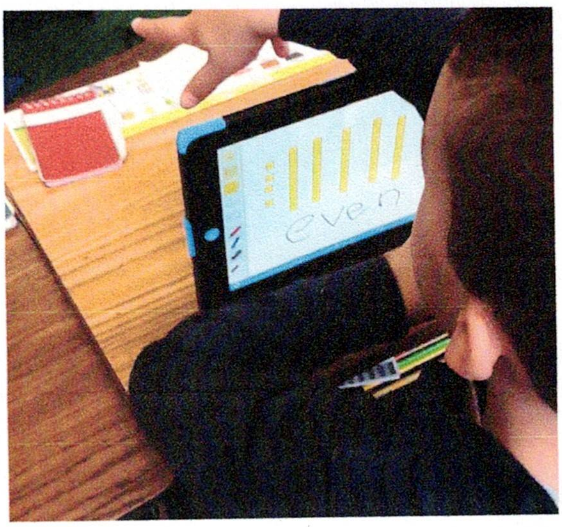
Warshauer (2015) describes the following four strategies to support students' productive struggle:

Strategy 1 – Teachers ask questions that help students focus on their thinking and identify the source of their struggle, then encourage students to look at other ways to approach the problem.

Strategy 2 – Teachers encourage students to reflect on their work and support student struggle in their effort and not just in getting the correct answers.

Strategy 3 – Teachers give time and help students manage their struggles through adversity and failure by not stepping in too soon or helping too much and thus take the intellectual work away from the students.

Strategy 4 – Teachers acknowledge that struggle is an important part of learning and doing mathematics.



> During my RISK Training, I watched children from other schools explain their thinking and assumed they were just "smarter" than the kids I work with. Now my students are those "smart" kids because they can explain and show their thinking! Thanks for teaching me as a teacher how to model thinking for open-ended questions. It feels as though teachers teach for the "right" answer versus the strategies, which far outweighs the answer for my students.
>
> — Tanya Smith, 2nd Grade Teacher, Washburn School, Maine

Teachers' responses to student struggles generally fall into four types (Warshauer, 2014a):

1. **Telling** – When using the telling strategy, teachers often suggest a new approach, correct an error, or supply information.
2. **Directed guidance** – Directed guidance involves redirecting student thinking by asking open-ended questions, breaking down the problem into smaller parts, and narrowing down what the student might try next.
3. **Probing guidance** – Probing guidance puts the struggle back into the student's lap. Here the teacher offers ideas based on the student's thinking, asks for an explanation that might surface an error, or asks for reasons and justifications.
4. **Affordance** – Affordance provides an opportunity for students to continue thinking with little help from the teacher other than encouragement.

All of these approaches are useful as long as the level of cognitive demand remains high, and student thinking is supported (Warshauer, 2011).

The following practices also help support student struggle and make it productive:

- Set goals at the beginning of the lesson and keep track of student progress during the lesson.
- Set problems in a familiar setting whenever possible, such as a sport or a familiar everyday task.
- Support students by providing appropriate tasks, tools, and representations.

- Group students heterogeneously, which helps struggling students.
- Establish high mathematical expectations (i.e., doing *mathematics requires effort)*.
- Use good questioning techniques, such as asking students to explain how they solved a problem and why a strategy works or ask them to describe another way to solve the same problem.
- Provide time for group reflection during problem-solving activities. This can help students recognize unproductive strategies.
- Compare student outcomes at the end of the lesson to your original goals.

Establish class norms that support productive struggle, such as these examples:

- Being wrong is an opportunity to learn.
- Being correct is an impetus to help others.
- Everyone is responsible for each other's learning.

Conclusion

"Productive struggle is complex. A student's effort to make sense of mathematics, to figure something out that is not immediately apparent can advance the students in their thinking and play an important role in deepening students' understanding, if supported carefully toward a resolution and given appropriate time" (Hiebert & Grouws, 2007).

Teachers need to carefully select tasks that require students to struggle and provide the support that students need without diminishing the cognitive demand of the task or giving students too much help. Students need sufficient time, not only to solve difficult mathematical problems, but also to "develop genuine curiosity and stamina" (Goldenberg, et. al., 2015). Finally, teachers must create a classroom culture that demonstrates "struggle as a natural part of the learning process" (Star, 2015) and allows students to see the potential in persevering.

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5 Principles of the Modern Mathematics Classroom – Creating a Culture of Innovative Thinking - Gerald Aungst. (2016)

This is hardly a new idea in math instruction. Japanese researchers have been studying the effects of an open-ended strategy since the early 1970s (Shimada, 1997). And while I often hear teachers talking about using open-ended problems, I rarely see the kind of messy problem solving that leads to sophisticated understanding of mathematical ideas.

PRODUCTIVE STRUGGLE

Understand that we are not talking about a free-for-all classroom. Perhaps a more precise description of the messiness we are looking for is *the edge of chaos*, a phrase coined by Christopher Langton, which Steven Johnson (2010) explains is "the fertile zone between too much order and too much anarchy" (p. 52).

Langton uses a metaphor of the phases of matter to help explain where this zone lies. The most chaotic state is a gas: molecules are in constant motion, and there is little hope of any patterns forming due to its extreme volatility. The opposite state is a solid: stable, predictable, and rigid. There are lots of patterns, but they are predetermined and unchanging. It's unfortunate that the word "rigor" tends to evoke this state, since little learning takes place here.

> The sweet spot in learning, this liquid state, is what Jackson and Lambert call "productive struggle."

However, in a liquid state, the molecules have the benefits of both extremes: there is lots of motion, but the motion can be contained and directed. New patterns can form, but they can also be retained.

The sweet spot in learning, this liquid state, is what Jackson and Lambert (2010) call "productive struggle." To understand this, let's look first at its opposite—Destructive struggle. This condition

- leads to frustration;
- makes learning goals feel hazy and out of reach;
- feels fruitless;
- leaves students feeling abandoned and on their own; and
- creates a sense of inadequacy.

It is certainly out of a desire to avoid these responses that we often feed students the algorithms and clues to getting to a solution. Don't be in too much of a hurry to bail them out, though. Let them get lost in the muck for a while. It's okay. By reasoning through and working it out on their own, they begin to better understand the territory where the problem lives. Mason, Burton, and Stacey (2010) tell us, "Probably the single most important lesson to be learned is that being stuck is an honourable state and an essential part of improving thinking" (p. viii). This is where productive struggle comes in. Productive struggle is a condition that

- leads to understanding;
- makes learning goals feel attainable and effort seem worthwhile;
- yields results;
- leads students to feelings of empowerment and efficacy; and
- creates a sense of hope.

This is the type of struggle that Hiebert and Grouws (2007) envisioned. Within that struggle, students also begin to understand their own thought processes and the kinds of thinking that lead to better solutions.

Brown (1993) refers to this state as "confusion," but I prefer to call it Chaos since in the classroom it is as much about the collective attitude and approach as it is the individual's state of mind. Teachers and students who can learn to tolerate the messiness inherent in productive struggle produce more significant results. Hiebert and Grouws (2007) report on a study by Inagaki, Hatano, and Morita that showed even when classroom discussions "were filled with confusions and incorrect conjectures along with correct analyses, most students made sense of the discussions and improved their understanding as demonstrated through both their verbal statements and written work" (p. 390).

Teaching Like Video Games

MP4, MP5, MP7, and MP8

Video games give us a nice model for building the kind of skill and confidence that allows students to engage in productive struggle. In video games, it is rare to have an instruction manual; players simply dive in. The early levels provide easy experiences designed to orient the player to the game world, introducing concepts one or two at a time. Yet, these levels are never merely exercises. Accomplishments make a difference in the game, and progress is always relevant to the overall game goals. In spite of the fact that they are typically simple and the stakes are low, the player never feels like he or she has to wade through practice material to get to the "real" game.

Another feature of these games is that the introduction of new skills is almost always done by putting the player into a real world (or in this case, game world) situation where the skill is necessary. Often, players are given minimal procedural directions and have to work out, in the context of the game, how to apply the new skill and ways to use it effectively. In early levels, there is often an obvious choice, but it is still left to the player to make it. The game quickly takes the player to novel situations where the obvious application of a technique may not work and the player has to reason his way to a solution.

Games respect the Chaos needed for the learning process by lowering the stakes involved in making mistakes. A character's death, for example, is rarely permanent, and even when it is, it is usually possible to go back to a previous checkpoint to try to regain a position. The ability to attempt the same puzzle over and over, learning from each attempt, just as Edison did, is one of the things that makes games fun and engaging.

Math class can be this way too. David Ginsburg promotes this in his work coaching teachers.

> How can we teach students to make sense of problems and to persevere when a problem is challenging? In my experience, we can't. We can, however, *create classrooms that cultivate these skills*, and the formula for doing this is simple: provide students challenging problems, and empower them with the skills and resources they need to solve those problems. Then get out of their way. (Ginsburg, 2014, para. 3, emphasis mine)

So what would a math class look like that is based on this video-game model?

First, provide students with a series of problems of increasing complexity. Begin with simple, straightforward problems where the solution is relatively obvious and the student can succeed easily. Quickly move on to problems that are more challenging, until students are working on very rigorous, deep, and open-ended problems around the same general topic. Keep the content challenging as well. Don't assume young children can't handle the material; advanced concepts and content should be a part of math instruction from the first day children walk into Kindergarten (Claessens, Engel, & Curran, 2014; Engel, Claessens, & Finch, 2013).

Two additional elements characterize the video-game approach from a more traditional one: feedback and pacing.

Two additional elements characterize the video-game approach from a more traditional one: feedback and pacing. After each attempt, successful or not, provide students with specific feedback about their performance. Briefly, point out areas they should examine again, places where they made errors, and opportunities for improvement. Guide them toward a more successful performance, but avoid pointing them directly to a solution.

The second element, pacing, is driven not by the curriculum guide, but by the student. In a video game, as soon as the player successfully demonstrates a skill, he or she can move on to the next level. In the classroom, as soon as the students successfully solve a problem, allow them to move on to the next level of complexity. While this takes additional planning and record keeping, and students need some help navigating the new process where they set their own pace, it is well worth the investment when you see students who are able to attack difficult problems with enthusiasm and self-confidence.

Avoiding Filters

MP3

It is tempting to come to a struggling student's rescue immediately. After all, it's in our nature to want students to succeed, and when we see struggle, we want to help. Remind yourself, though, that too much help is no help in the long run. It creates dependence and prevents growth.

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Instead of pointing students directly to the correct solution path when their first attempts fail, initially focus on what worked, then on *why* their solution did not ultimately succeed. Suggestions are always better than corrections, and use thought-provoking, reflective questions instead of leading questions. For example, in a multi-step problem where the student selected a good strategy, but made a computation error, instead of pointing out the mistake, or asking whether they checked their work, try the following series of questions:

What kinds of errors could have caused this? Which one of them is most likely for you? Which one is the simplest to find and fix? Try looking there first. Of course, it is entirely possible that they select and look for the wrong kind of error. Let them. When they struggle, come back and ask further questions: Did that path work? What other kinds of errors did you think of? Did you try looking for those? Are you keeping track of the areas you're checking so that you don't have to go back and look again?

You may be thinking that it is better to give the student a checklist of steps to follow when solving and checking errors in their problems. Again, resist this urge. Let the student discover that they are going through the same steps over and over, and that it would be helpful to have a checklist. Then, either help them to create their own (which is the most powerful option), or offer to provide one that you found helpful to your own problem solving. Either way, now each student will internalize the value and application of the checklist in a way not possible if you had provided it first.

I suspect you may also be writing some objections in the margins. *This is all well and good, but it's not how things work in the real world.* Leaving aside the question of how much and how soon the process of learning needs to precisely mirror the world beyond school, let's consider how product development works at one of the world's largest and most successful companies, Apple Inc.

In a traditional manufacturing process, each stage is handled by a separate team. The design team begins by developing the concept for a product. The engineering team then determines how to make it work. The manufacturing team works out the processes for building it, and then the marketing team figures out how to sell it and how people can buy it. "This model is so ubiquitous because it performs well in situations where efficiency is key," says Steven Johnson, "but it tends to have disastrous effects on creativity, because the original idea gets chipped away at each step in the chain." For example, engineering decides they can't make the entire product work as designed, so they filter out the parts they can't do. Manufacturing takes out some other elements that are too complex or too expensive to mass-produce.

Apple, on the other hand, puts all of these teams in one room and works on all stages at once.

> The process is noisy and involves far more open-ended and contentious meetings than traditional production cycles—and far more dialogue between people versed in different disciplines, with all the translation difficulties that creates. But, the results speak for themselves. (Johnson, 2010, p. 171)

The bottom line is that Apple's approach, just like the Chaos principle, avoids limiting thinking and promotes creative problem solving. Students learn to honor each other's ideas. They also learn to have confidence in their own brainstorms, and eventually to be able to assess and revise them, without intervention by experts. Once they get to handle truly unsolved problems, where there is no predefined solution and no expert to guide them, they are able to jump in and persist until they arrive at a useful solution. Don't sacrifice quality learning for efficient teaching.

GRADES K–3

The Principle of Chaos may seem simple to achieve in the primary grades, due to the high energy level of most K–3 students. Teachers can take advantage of the natural Chaos in their students, just as they use their natural