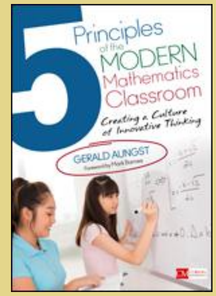


File: Teaching Mathematics

5 Principles of the Modern Mathematics Classroom:
Creating a Culture of Innovative Thinking

By Gerald Aungst (Corwin Press, 2016)

S.O.S. (A Summary of the Summary)

The main ideas of the book:

~ We must create mathematics classrooms in which problem solving occurs daily and is deeply embedded in the culture.
~ The 5 Principles presented in this book will guide teachers in creating a modern mathematics classroom that supports this type of culture.

Why I chose this book:
This book is not about math content. Nor is it a book with tips or cool math activities. Instead it paints a larger picture of the five principles needed to form the foundation of the modern mathematics classroom. It aims to reshape the culture of more traditional mathematics classrooms into a place where students deeply engage in problem solving.

The book is appropriate for math teachers at all levels, K-12. In addition, it provides a helpful tool for the school leader to know what to look for when observing math teachers.

I like that the book is easily digestible even for the most math-phobic school leader. Further, this book can be used in conjunction with any existing math program and is applicable whether or not your school or district follows the Common Core State Standards.

The book includes practical strategies and applications and also suggests how technology and 21st century skills can be integrated into any math class at any level.

Gerald Aungst is a district supervisor of gifted education and elementary mathematics after teaching mathematics at the elementary level for eighteen years.

The Scoop (In this summary you will learn…)

✓ The five principles that must form the foundation in any modern mathematics classroom:

- Conjecture – Students engage in inquiry, questioning, and problem finding
- Communication – Students read, write, speak, and listen as they formulate and support mathematical arguments
- Collaboration – Students work in pairs and groups to support, encourage, and help each other
- Chaos – Class is understandably messy when students are truly allowed to struggle with mathematical concepts
- Celebration – The focus is on effort over achievement and small wins are celebrated

✓ Professional development suggestions from THE MAIN IDEA to introduce the ideas in the book to your math team

Introduction, Chapter 1 and 2: Problem Solving and the Modern Mathematics Classroom

This book is not about math content. Nor is it a book with tips or cool math activities. Instead it paints a larger picture of the five principles needed to support the modern mathematics classroom. It does provide practical applications and also suggests how technology and 21st century skills can be integrated, but it aims to reshape the culture of our mathematics classrooms today.

A Culture of Learning and Problem Solving in the Mathematics Classroom
How often do we hear adults comfortably say, "I'm just not that good at math" when they would never express such a sentiment about reading or social skills. We live in a culture in which math is seen as a specialized skill only some people are born with. Unfortunately, we often reinforce this notion in our math classes. We teach math as if there were only right answer and there is no room for innovation or creativity. This book provides an alternative in which math is taught as a framework for problem solving and reasoning rather than as a set of rules to memorize. Students need to learn to see math problems in a variety of ways and have the confidence that they can solve those problems. Instead, we teach in a more restrictive way and promulgate the idea that students must learn the "one right way" rather than develop more free-range thinking. Although the Common Core State Standards carry political baggage, they do provide the opportunity to help us rethink our practices. If we can use these standards as a springboard to infuse our math classes with problem solving and the thinking that supports it, this will be an impressive step in the right direction toward improving the mathematical skills of our students. This book introduces five principles that will help mathematics teachers make fundamental changes in the culture of learning in their classrooms: Conjecture, Collaboration, Communication, Chaos, and Celebration. Chapters 3 to 7 will each explore one of these principles and help teachers think about ways to implement them.

For book study groups
• Without knowing the specifics of each Principle, which do you think you would be most comfortable with, which do you think would give you the most anxiety, and why? (Conjecture, Collaboration, Communication, Chaos, and Celebration)

A New Culture for the Modern Mathematics Classroom
Problem Solving and Rigor
One of the obstacles in creating a rigorous mathematics classroom centered on problem solving is that we often confuse giving students math "problems" with math "exercises." Take a look at the following two examples:

> Miguel collects baseball cards. Last week he had 217 cards in his collection. Today, his aunt gave him two dozen more for his birthday. How many cards does he have now?

> You and your friends are going to play a game using a set of cards numbered from 0 to 9. On your turn, you are going to draw three cards from the facedown deck, one at a time. The object is to make the largest 2-digit number you can using your cards, with the leftover card being discarded. The catch is that you must decide where to write each digit before you draw the next: tens place, ones place, or discard. If you draw a 4 as your first card, where should you write it, and why?

The former is a typical textbook problem at the elementary level. While it does require students to complete a few steps, it is a fairly straightforward process that involves applying the skill of adding multi-digit numbers to one concrete problem. The second problem is not as simple. It involves more reasoning and conceptualizing about the relationship between numbers. To further understand the difference between the two, Wikipedia defines an "exercise" as "a routine application of ... mathematics to a stated challenge. Teachers assign mathematical exercises to develop the skills of their students." The important word here is *routine*. Like when students practice scales in music, through repeated practice with a math skill, students develop fluency and automaticity. However, this is not a complete performance. If we want students to develop a deeper understanding of math concepts, we need to consider the *rigor* of the problems we are giving. One useful tool to do this is Norman Webb's Depth of Knowledge. Take a look at Webb's descriptions of the different levels of thinking along with the types of math problems that fit each level:

Webb's Depth of Knowledge (DOK)	Example from Area and Perimeter	Example from Quadratics
Level 1: Recall and Reproduction (recalling basic facts)	Find the perimeter of a rectangle that measures 4 units by 8 units.	Find the roots of the equation: $y = 3(x-4)^2 - 3$
Level 2: Skills and Concepts (involves some decisions and skills such as comparing, organizing, and estimating)	List the measurements of 3 different rectangles that each has a perimeter of 20 units.	Create 3 equations for quadratics in vertex form which have roots 3 and 5, but have different max or min values.
Level 3: Strategic Thinking (involves planning, evidence and more abstract thinking – such as solving a non-routine problem or explaining the reasoning behind a Level 2 problem)	What is the greatest area you can make with a rectangle that has a perimeter of 24 units?	Create a quadratic equation using the template below with the largest maximum value using whole numbers 1 to 9 no more than once each: $Y = -\square(X - \square)^2 + \square$
Level 4: Extended Thinking (synthesizing information over an extended time, transferring knowledge from one domain to another – such as designing a survey and interpreting results, analyzing multiple sources of raw data or solving problems with no clear solution)	There is no example in the book.	There is no example in the book.

Overview of the 5 Principles of the Modern Mathematics Classroom

Using Webb's Depth of Knowledge will help the math teachers at your school have a common vocabulary to increase the "rigor" of mathematics instruction beyond simply completing *exercises*. A classroom based on the 5 Principles should emphasize Level 3 problems as much as possible. Schools all over the U.S. are also now using the Common Core State Standards to improve the rigor of their math instruction. Aungst believes the most important part of the CCSS is the Standards for Mathematical Practice. Unfortunately, even though this section is at the *front* of the document, the practices are not embedded in the content standards. However, the 5 Principles encompass all eight of the Practices, and you will see them woven throughout the strategies presented in the book. In fact, the author argues that when the Mathematical Practices are taught in isolation, this is no different than teaching definitions and algorithms in isolation. If we want students to fully develop the Practices, we need to create an environment that will continually support them. Such an environment should have the following 5 Principles:

5 Principles	Traditional Classroom	Modern Mathematics Classroom
Conjecture	The goal is for students to get the right answers to questions and exercises.	Students ask most of the questions and conjecture is encouraged. The answer to a question is often another question. Inquiry and problem solving are valued.
Communication	Communication is one way with the teacher explaining a procedure or algorithm to the students.	Students communicate frequently about problems and how they solve them. They develop their writing, vocabulary, and metacognition. The focus is on formulation and support of mathematical arguments.
Collaboration	Students work alone and the focus is on each individual's skill fluency.	Group work is more prevalent than individual work and students are encouraged to share ideas and answers and ask for help. In a problem-solving culture, students are cheerleaders for each other, not competitors.
Chaos	Neatness and order are prioritized. Students learn a procedure then replicate it flawlessly.	Real problems are messy – they involve experimentation, false starts, mistakes, and corrections.
Celebration	Recognition is for right answers and high grades.	Anything that moves *toward* a solution is celebrated, including small steps in a complicated problem, and finding an innovative approach. Effort is rewarded over achievement.

For book study groups

- How does Webb's Depth of Knowledge help you think differently about classroom tasks and assessments?
- Keep a list of math tasks you give students in one day (or one week). Sort these into the four DOK levels. What patterns do you see? What did you learn about your practice? Take a Level 1 or Level 2 task you gave and *rewrite* it to be a Level 3 task.

Chapter 3 – The 1st Principle: Conjecture

Overview of the Principle: In the modern mathematics classroom, Conjecture becomes a regular part of the culture as students develop this habit of mind. The focus is on questions rather than answers and one true path. Human brains are naturally wired to wonder, but as teachers we take away student curiosity by laying out all of the steps students will follow. Imagine if we presented a mystery novel to students and started by telling them who committed the crime!? Rather than demonstrating a math strategy for students and then marching them through the steps, there is a simple fix: pose a compelling problem and *first* let the students grapple with it. This does not mean simply handing over a problem to your students, but by starting with a *problem*, rather than an isolated strategy (like factoring polynomials), we provide the context and motivation for students to dive in. Below is a way we can teach students that solving problems is more like a *cycle* than a recipe:

Step 1. Recognize or identify the problem.
Step 2. Define and represent the problem mentally.
Step 3. Develop a solution strategy.
Step 4. Organize knowledge about the problem.
Step 5. Allocate mental and physical resources for solving the problem.
Step 6. Monitor progress toward the goal.
Step 7. Evaluate the solution for accuracy.

The problem is that in the traditional math classroom we often have students focus almost exclusively on finding the right answer – steps 5 and 6 – while other people (the teacher, the textbook author) identify the problem, present the most efficient strategy, organize the information, and more. Instead, teachers should find intriguing problems and allow students to ask the questions that allow them to explore the problem. A few ways to approach this are below and more are in the book. Each chapter provides several suggestions for how to approach the Principle for different grade bands as well as digital tools you can use to support the Principle.

Grades K-5: Some strategies to try with younger students are *Never End With the Answer* and *Always Ask Why*. Rarely should an answer stand on its own in a math classroom. Below are some sample questions you can use. Further, these will add depth to any textbook problem. While students may be annoyed at first, with practice they will start to ask these questions of themselves. The first series of questions are best with K-3 students and the second set are for grades 2-5:

Questions for Grades K-3	Questions for Grades 2-5
Why do you think so? How did you get that answer? Are there other ways to answer it? What was hard about solving that problem? What did you use to help you solve this?	What about this problem feels familiar? Why? Why do you think this works? Does it always work? What isn't working? Why? Does anyone want to add to the solution?

Middle and High School: For older students, patterns provide an excellent opportunity for Conjecture. Consider using tessellations or patterns in other areas such as weather, writing, or current events. High school students should be looking at more complex problems that do not have an obvious solution. One example in the book that stumped high school students involves figuring out how to use a box of tacks, a book of matches, and a candle to light and hang on a wall without the wax dripping (see the book for the solution!) The best way teachers can support students in solving problems – give them lots of problems to solve.

Digital Tools: To help students verbalize their reasoning, you can use whiteboard apps such as **Educreations, ShowMe, PixiClip**, or **ScreenCast** to make and share recordings of students working on problems. To find compelling problems that will encourage Conjecture, see the website **MathPickle**. **WeLearnedIt** is a social learning platform for students doing project-based learning.

For book study groups

- Does your math instruction more resemble a recipe or a cycle? Are there steps you often do *for* the students?
- How might you balance a culture of Conjecture with preparation for state tests?

Chapter 4 – The 2nd Principle: Communication

Overview of the Principle: Math is not simply about computation and skills. It is about solving problems and communicating how they are solved. In fact, most math learning takes place when students are talking to each other and writing. For this reason, the math classroom should become a Communication classroom filled mostly with talking and writing every day. Students need to explain, argue, defend, critique and discuss mathematical ideas. To do this, the focus should be on vocabulary, writing, and the formulation of mathematical arguments. Students need to go beyond math symbols and be able to discuss and write about math using English. In order to truly learn math concepts, students must progress to Level Six in the framework from David Sousa below – that is, students must be able to explain what they have learned:

	Explanation of the Level of Mastery	Illustration of Each Level
Level One	Connects new knowledge to existing knowledge	Student recognizes fractions are related to division and ¾ is the same as 3 ÷ 4.
Level Two	Uses concrete material to construct a model of the concept	Student measures 3 cups of sand, then divides it into 4 equal piles so each contains ¾ of a cup.
Level Three	Illustrates the concept by drawing a diagram, symbolic picture, or representation	Student draws picture showing $3.00 can be divided into 4 equal amounts by exchanging dollars for quarters.
Level Four	Translates the concept into mathematical symbols	Student writes ¾ = 3 ÷ 4 and $3.00 ÷ 4 = $0.75
Level Five	Applies the concept correctly to real-world situations or story problems	Student solves, "Elana's band wants to hold a 3-hour dress rehearsal and has 4 songs to practice. How much time should they practice each song if they want to spend the same time practicing each song?"
Level Six	Can teach the concept successfully to others or can communicate it on a test	Student explains orally to a peer how fractions and division are related and then explains her answer to the band problem above.

Grades K-5: For younger students, explicitly teach and use academic vocabulary. Instead of asking for the "answer," ask for the *sum, difference, product,* and *quotient.* Have students deepen their understanding of math terms by explaining them in their own words, sorting them, discussing them, and playing games with the vocabulary terms. Make sure they can write and understand terms written in different forms. For example, 728 is also 700 + 20 + 8 and seven hundred twenty-eight. Give students the type of longer problems that invite casual conversations rather than structured ones. For example, give students a small box and a large piece of paper and ask how they could cut out the paper to cover the box with one piece (working with Nets). Rather than discussing with the whole class, join small groups working on the problem and model asking questions such as: *What is confusing about this problem? What if you tried the opposite of what you are doing now? Are you missing any information? What's the craziest idea you have to solve this?*

Middle and High School: Give students daily opportunities to write, using: quickwrites, individual whiteboards, exit tickets, math fiction, and math research. Have students work to convince people of their ideas – first they should convince themselves, then a friend, then an enemy. Don't let students get away with just one solution. Have them explain it in another way or work in groups to come up with two or three different explanations

Digital Tools: Digitally record students solving math problems with tools. Have students watch their own explanations and reflect on how they can explain better. In addition, have students create instructional math videos for younger students. To support online conversations about math try **Edmodo** and **Google Classroom**. To give students an authentic audience for their communication, have students create blogs or podcasts.

For book study groups

- Review the six levels of mastery in the chapter. How many could you say your students demonstrate?
- For elementary teachers: How could you use reading, writing, speaking, and listening to foster problem solving throughout the day? For secondary teachers: How could you broaden your knowledge about language arts to improve your teaching of these skills in math?

Chapter 5 – The 3rd Principle: Collaboration

Overview of the Principle: In real life, we rarely have to solve problems alone. And the idea that getting help from colleagues is "cheating" is ridiculous. So, we need to change the culture of our math classrooms so that Collaboration becomes the norm. In a traditional classroom, the focus is on developing the skills of individual students. Instead, we need to recognize that collaborative learning is not only what is required in the real world, but it also contributes to individual learning. By working with other people, students are required to provide explanations, ask questions, listen, respond to the perspectives of others, give feedback, and reorganize their own thinking because of exchanges with others. Students would not practice all of these skills if they only engage in solo work. In fact, it could be argued that the majority of the work in a math class should be tackled as a "we" and only after working in pairs or groups should students embark on tackling a problem by themselves.

Grades K-5: Younger students need more structure and more explicit instruction in how to work collaboratively. One approach to try is the Think-Play-Pair-Share strategy. This is similar to what math educator Magdalene Lampert calls "You, Y'all, We." In this approach students *begin* with individual exploration, then move on to groupwork and then whole class discussion. In the first stage, *Think*, students are given a problem to consider for a few minutes and possibly some guiding questions (*What do we know? What do we need to know? What could help us get the answer?*) In the second step, *Play*, students are given plenty of time and little prompting to experiment with the problem with no expectation of solving it. In the *Pair* step students attempt to find the solution together with a partner. In the *Share* stage pairs share their work with the class. At this point the teacher can introduce algorithms for efficiency.

Middle and High School: In older grades students should take more ownership in finding and creating problems. In groups, students can be given challenges such as: Create a problem with at least two different, yet correct answers. Or, given an infographic, create a problem. In the same way teachers hold writing workshop, they can do the same in math. Once a week you can introduce interesting problems to solve and give students the time and space to work on those problems in small groups. Math classrooms become even more conducive to collaborative problem solving as students get older and problems become increasingly complex. Consider asking students to solve something like this: The lockers at our school are arranged by last name rather than by proximity to classes. How could we reorganize the lockers to help as many students as possible carry a heavy backpack the least distance possible? Students could share their hunches on paper or digitally before diving in. A simple way to make Collaboration a daily routine is to grab an interesting statistic from the news, a book, or a magazine (a list of data sources is on p.90 of the book) and have students work in groups to analyze it, interpret it, share their thinking about it, and present it.

Digital Tools: To connect students with other students, you can use **Skype** (see the Skype in the Classroom project). To allow students to share and edit information, see **Google Drive** and **Padlet.** To help students compile, access, and organize their hunches to a solution, try **Tricider** or **Trello**. To help students collect and discuss their data, try **Evernote** or **Simplenote.**

For book study groups

- What are some ways to ensure individual accountability for mastery of skills without sacrificing the culture of Collaboration?
- What are some daily routines you can use to establish a collaborative environment?

Chapter 6 – The 4th Principle: Chaos

Overview of the Principle: One of the best ways to help students develop their *conceptual* understanding of mathematics is to allow students to *struggle* with mathematical concepts. For students to get better at problem solving, they need to be immersed in an environment that is a bit messy. We're not talking about the kind of chaos that arises from poor classroom management or utter disorder, but rather the kind of atmosphere in which students are allowed to have false starts, reversals, and abandoned trails as they attempt to solve problems. Instead of structuring class so students follow along, step by-step, learning to solve one particular problem that someone else has already figured out, and then replicating and rehearsing those steps, we need to help students become better at solving *all* problems. This happens when the class has a bit more noise and error. When students and teachers can tolerate Chaos that comes from productive struggle, they will see better thinking. A good model for this is video games. Students learn to play them without an instruction manual. They dive right into the first few levels without practicing isolated skills first. It's slightly chaotic as students fail, over and over, in an attempt to meet the challenges. However, the stakes are low so even if their character "dies," they often have additional chances or can just play again. We can set up something similar in math class. For example, we can provide open-ended questions that allow for messiness and failure, give students plenty of time to experiment, and keep the stakes low. We can start with easier problems and gradually move students toward more complex ones. Like with video games, we can provide feedback without pointing students directly to a solution. It is tempting to want to rescue a student who is struggling, but this ultimately impedes growth. Instead, ask thought-provoking (not leading) questions: *What kinds of errors could have caused this? Which is the simplest to try to fix?*

Grades K-5: One strategy to move students away from a focus on the right answer is to supply the answer yourself! This way the focus can be on the *process* – what they did to solve the problem -- rather than just getting the right answer. It is also important to introduce non-routine problems so students are not just looking to find the right algorithm. For example, tell students the grid below represents 16 rooms, each of which is connected to the rooms around it and must be painted. A painter needs to start in room 1 and end in room 16, but he cannot return to a room once he has painted it (the floor is wet!) What path can he take? Try it yourself. It's impossible if he starts by painting the floor in room 1. But who said he MUST paint the floor in room 1 *first*? This is an example of a non-routine problem. The websites below offer examples of non-routine problems you can use. These non-routine problems remind us that instead of teaching students algorithm after algorithm, we should teach students *heuristics* – problem-solving strategies (such as: draw a picture, work backwards, guess and check, and in this case, re-example your assumptions).

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Middle and High School: By middle school, students are more ready to conduct their own investigations. Consider doing something like Google's "20% time" in which they allow employees to use one-fifth of their time to experiment with their own ideas. Set aside time each week for students to tinker with a problem or mathematical question they choose. You can bring them to a new setting – to the auditorium, outside, or just allow them to sit on a comfortable couch – to spark their thinking (the same way some of us go to Starbucks to change the setting). Take note of the mathematical skills and concepts they are applying. Get them used to working with more complex problems by giving them sample problems that *you* do not know the answer to or that have no answer. Model what it looks like to wonder about something (the author calls these iWonders), "iWonder what is the weight of all the asphalt and concrete on I-95?" "iWonder if we took all the eggs laid by all the chickens in Iowa on May 4 how big an omelet it would make?" You can also tweak traditional textbook problems to make them more intriguing (see examples of "Makeover Monday" here: **http://blog.mrmeyer.com/category/makeovermonday/**) Still another way to have older students work on more complex activities that allow for some messiness is through computer programming.

Digital Tools: For non-routine problems, there are websites on p.112 such as: **Bedtime Math, Dan Meyer's Three Act Problems**, http://www.math.com/teachers/recreational.html, http://www.insidemathematics.org/problems-of-the-month (problems of the month organized by CCSS), http://math.com/teachers/POW.html (problems of the week). For websites with information to spark iWonder problems: **uselessfacts.net, Listverse, Worldometers, Mental Floss, tylervigen.com.** To introduce computer programming languages, try: **Scrach** or **Alice**.

For book study groups
• Chaos looks different from what principals and parents expect to see. How can you help them understand what you hope to do?

Chapter 7 – The 5th Principle: Celebration

Overview of the Principle: Celebration is so important in the math classroom that it should occur *daily*. It should be part of every interaction, every piece of feedback, and every activity. The reason to incorporate Celebration is that it helps students believe that growth and improvement are not only important, but possible. When students have what Carol Dweck calls a "fixed" mindset, they believe their intelligence is fixed and effort won't help. However, if we create a culture in our classrooms in which failure simply means "not yet" and effort is rewarded, students' mindsets can change. With the right encouragement they can develop a growth mindset and start to believe that effort is productive. This is where Celebration comes in. By celebrating what students have done well, and guiding them toward where they still need to go, we can help students develop this growth mindset. This is not the type of Celebration in which everyone gets a trophy and flaws and mistakes are ignored. Instead, Celebration must be authentic. We need to create a culture in which mistakes and failures are a natural part of class, but students *learn* from those mistakes. In fact, Dweck reports that students who believe their lack of effort leads to their mistakes are *more likely* to keep trying when problems get tough. To incorporate the type of Celebration that fosters a growth mindset, make sure to: (1) celebrate and share the thinking and learning rather than the product and, (2) highlight the perseverance and struggle rather than cover it up. Another suggestion for a classroom that focuses on Celebration is to eliminate grades entirely and replace them with meaningful feedback. By removing the pressure of grades, you can help students focus on learning and growth by providing feedback on what students have accomplished and what they still need to understand, and allowing them to resubmit work that needs to be improved.

Grades K-5: Younger students are particularly susceptible to the idea that you are either right or wrong. To help them appreciate the *process* and get through the bumps of a tougher problem, make sure to celebrate small wins along the way – this can be a simple high-five, an announcement to the class, or publishing their photograph on the class blog. Another idea for upper elementary students is to give them the opportunity to do a "daily edit." Each day give them the solution to a math problem that contains errors. You can create it yourself, use examples from students with their permission, or find examples from this website by topic or Common Core standard: **http://mathmistakes.org/.** Then have students look for the errors. This will help normalize the occurrence of errors, show that making errors is not disastrous, and teach them to improve their ability to look for errors in their own work.

Middle and High School: You can do something similar to the "daily edit" above and it will have the same benefits. Tell students you are going to make deliberate mistakes in class and it's their job to catch them. Another idea for the upper grades, since teachers often have students present solutions to the class, is to have students give their solution a *certainty rating* (1 = I have a good start, 2 = I'm confident, 3 = I guarantee it, ? = I'm not ready yet). These ratings will help you know what kind of feedback, praise, and encouragement to give to students. This also helps students monitor their own growth and learning.

Digital Tools: Many of the previous digital tools in this book can be applied to feedback and celebration. In addition, one tool for specific feedback is **Kaizena.com**. For celebration, use digital badges (see **classbadges.com, credly.com**, and **openbadges.org**).

For book study groups
• A focus of this chapter is turning mistakes into growth opportunities. How does your class support this approach?
• In contrast, which school or district practices undermine a culture of Celebration? How can you promote Celebration in your school?

Chapter 8 – Becoming a Problem-Solving Classroom

As you are well aware, a shift in your math class won't happen just by reading this book! Instead it involves a concerted effort in three areas: (1) self-assessment, (2) planning, and (3) maintenance.

Self-Assessment – To begin, you need to assess how well your classroom culture supports problem solving. First, brainstorm all of the routines, procedures, habits, norms, strategies, techniques, and expectations in your class. Write each one on an index card. Then sort them into five groups based on which of the 5 Principles they most support. Starting with one Principle at a time, divide the index cards into the practices that: (a) promote, (b) unsure/both, or (c) erode a problem-solving culture. Based on these three categories, reflect on which of these strategies you wish to keep, eliminate, or revise. Then do this with the other four Principles. To help you do a more in-depth job of assessing your problem-solving culture, have an observer come and look at the following three areas while observing for all 5 Principles in action:

- Teacher Verbal and Nonverbal Cues – Do they promote or erode a problem-solving culture?
- Student Responses – Do they show that problem solving is integral to this class?
- Daily Routines – Do these promote or erode a problem-solving culture?

Planning and Maintaining – Look at the patterns that emerge from the data you collected about your class culture above. Use this data to choose one Principle to work on first and create an action plan. The action plan should take into consideration everything that may affect the culture -- from the physical layout of your classroom to the assessment practices. Below is a chart to map out your action plan:

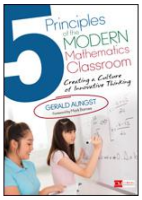
Area of Focus	Action Steps	Time Frame	Person Responsible	Resources Needed
Physical space				
Curriculum materials and resources				
Instructional practices				
Assessment practices				
Daily routines and procedures				
Social/emotional environment				

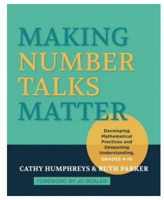
For each area of focus, choose one *overarching goal* and one *subgoal* for the Principle you've chosen. For example, for Conjecture, the overarching goal might be that by the end of the first quarter *students* will ask at least half of the questions in a typical lesson and students will have the confidence to answer each other's questions. Then think about how the class *layout* and *materials* would support this goal. Further, how could your *instructional strategies* support this goal? Keep in mind that changing your classroom takes time. Focus on one Principle at a time for about three or four weeks before moving on to the next one. Then to *sustain* those changes, find a colleague to check in with and share ideas. If there isn't anyone at your school working on these issues, find other teachers through a PLN or join the community of teachers discussing this book. After a year of implementing the 5 Principles, your students will be better problem solvers and more innovative thinkers!

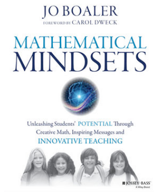
THE MAIN IDEA's PD Suggestions for Math Teachers

Note that these professional development suggestions are for use with these 3 books:

(1) 5 Principles of the Modern Mathematics Classroom, (2) *Making Number Talks Matter,* and (3) *Mathematical Mindsets*







I. LEARN the philosophy underlying THREE books for math educators

A. Teachers Learn the Philosophy in One of the Three Books About Mathematics Education

1. Using a jigsaw approach, divide teachers into three groups and have each group learn about the philosophy in ***one*** of the three books below. While it would be ideal to have teachers read the entire book, given time constraints, teachers can focus on the chapters that contain the underlying philosophy of each book indicated below:

- Gerald Aungst's *5 Principles of the Modern Mathematics Classroom* (chapters 1-2, pp.1-20)
- Cathy Humphreys' and Ruth Parker's *Making Number Talks Matter* (chapters 1-3, pp.1-24)
- Jo Boaler's *Mathematical Mindsets* (chapter 9, pp.171-208)

2. Next, give teachers the chart below with an overview of each author's philosophy. Using the jigsaw approach, have the teachers who read each book introduce the ideas in that book. After all three groups have done this, have the larger group look for what is ***common*** among the three (for example, all believe in the importance of having the right mindset about math).

Humphreys & Parker	Aungst	Boaler
1. Be comfortable with wait time. 2. Encourage students to explain concepts (why) not procedures (how). 3. There is no one right approach and mistakes are opportunities to learn. 4. Learn to listen. 5. Mathematical understandings will develop over time. 6. Help students express themselves more clearly. 7. Get students to talk to one another. 9. Confusion and struggle are natural and necessary to learn. 10. Create a culture where all students feel safe to share their mathematical ideas.	1. Conjecture - Students engage in inquiry, questioning, and problem finding 2. Communication - Students read, write, speak, and listen when reasoning mathematically 3. Collaboration - Students work in pairs and groups 4. Chaos - Class is understandably messy when students truly struggle with mathematical concepts 5. Celebration - The focus is on effort over achievement	1. Everyone can learn math to the highest levels. 2. Mistakes are valuable. 3. Questions are really important. 4. Math is about creativity and making sense. 5. Math is about connections and communicating. 6. Depth is much more important than speed. 7. Math class is about learning, not performing.

3. Conduct a group brainstorm about how a classroom with the mathematical approaches described above would differ from a more traditional math classroom.

Traditional mathematics classroom	Classroom that follows the approaches in the 3 books
Ex. The goal is for students to get the right answers and get them quickly. Ex. Communication is one way with the teacher explaining a procedure or algorithm to the students. Ex. Students mostly work and are assessed individually.	

II. DO some of the math problems that reflect these new approaches

The best (and most fun!) way for math teachers to truly learn the ideas in these books is to have them actually *do* some of the math problems using these new approaches.

A. Participate in a Number Talk as if the Teachers were the Students (Making Number Talks Matter)

1. If you have someone who already has experience with Number Talks or would like to take the lead, read one of the chapters in *Making Number Talks Matter,* and act as the teacher, then you can have that person facilitate this section. If not, simply ask teachers to do the following problem, *in their heads*, without using the traditional algorithm (as a reminder, Number Talks can work with students at all ages as you will see college students discuss the problem below in a video in Step 3!):

18 x 5

2. Next, have teachers share all of the different methods they used to solve this problem (18 x 5) and have the facilitator record (using pictures if possible) all of the different solutions.

3. Have teachers watch and then discuss this video of Jo Boaler introducing Number Talks, Stanford students solving this problem (18x5), and explaining their thinking. (The video is 15 minutes, but you can share just the first 7 minutes): https://www.youcubed.org/from-stanford-onlines-how-to-learn-math-for-teachers-and-parents-number-talks/

B. Experience the Difference Between an "Exercise" and a "Problem" (5 Principles of the Modern Mathematics Classroom)

1. Create sets of cards from 0 to 9 or use decks of cards (the Jack can be the 0). Then divide teachers into pairs and have them play this game. After playing it several times, have them describe the strategy they used to win:

> You and your friends are going to play a game using cards numbered from 0 to 9. On your turn, draw 3 cards from the facedown deck, one at a time. The object is to make the largest 2-digit number with your cards, with one card being discarded. The catch is you must decide where to put each digit before drawing the next: tens place, ones place, or discard. If you draw a 4 as your first card, where should you write it, and why?

2. Next have teachers individually solve the following:

> Miguel collects baseball cards. Last week he had 217 cards. Today, his aunt gave him two dozen more. How many cards does he have now?

3. As a group, discuss how the two math tasks differ. During the discussion, share that Aungst calls the first one a 'problem,' and the second an 'exercise.' Wikipedia defines an exercise in this way:

> An "exercise" is "a routine application of ... mathematics to a stated challenge. Teachers assign mathematical exercises to develop the skills of their students."

4. Ask math teachers to come to future math team (or grade level) meetings with a new problem each time for the teachers to solve. Bring a calendar of all future meetings, sign up teachers, and provide them with these resources to help them find problems:
Bedtime Math, Dan Meyer's Three Act Problems, http://www.math.com/teachers/recreational.html, http://www.insidemathematics.org/problems-of-the-month (problems of the month organized by CCSS), http://math.com/teachers/POW.html, and more.

C. Have Teachers engage in a Rich Mathematical Task (Mathematical Mindsets)

1. Have teachers solve and discuss one of the problems in Chapter 5 of Boaler's book. A few that only involve pencil and paper are:
- How many ways can you create a rectangle with an area of 24?
- Can you make all of the numbers from 1 to 20 using four 4's and any operation. For example:

$$\sqrt{4} + \sqrt{4} + \frac{4}{4} = 5$$

2. Share Boaler's 6 ways teachers can adapt math tasks to boost conceptual understanding and engagement. Discuss what each of these mean and have teachers share ways they have included any of these design elements in math tasks they've assigned.

> 1) Open the math task to include multiple pathways (e.g., "You know the rule for $1 \div \frac{2}{3}$. Now *make sense* of your answer.")
> 2) Make it an inquiry task (e.g., Instead of find the area of a 12x4 rectangle, ask how many rectangles you can find with an area of 24.)
> 3) Ask the problem *before* teaching the method (e.g., Ask calculus students to find the volume of a lemon *before* teaching them how to find the area under a curve.)
> 4) Add a visual component (Have students draw diagrams, pictures, or use objects like multilink cubes and algebra tiles.)
> 5) Make the floor low and ceiling high (Give a problem everyone can solve but extend it by asking those who finish to create a new question that is similar but more difficult.)
> 6) Require students to convince and reason (Require that students give more information than just an answer on its own.)

Ask teachers to look at the rectangle or fours problem they solved above and discuss the design features included in this problem. (Note that not all math problems need to incorporate all 6 of these features to be good problems!)

III. TEACH (or plan to teach) math using a new approach

A. Have teachers create and role-play conducting a Number Talk

1. First, have teachers choose a topic (subtraction, addition, multiplication, division, or fractions/decimals/percents) to plan a Number Talk for. Then have them read the corresponding chapter in *Making Number Talks Matter*.

2. Next, have teachers create a 15-minute Number Talk by choosing a problem (and they can use one from the chapter), anticipating possible strategies students might use for solving the problem, and recording different strategies they think students might use. (Yes, they should actually write these out.)

3. Then have teachers role-play conducting a Number Talk with other math teachers who will act as students. Have one math teacher not participate as a student and instead observe and look for the following in the chart below. Debrief after the role-play, then give everyone a chance to role-play a teacher conducting a Number Talk.

1. Be comfortable with wait time. 2. Encourage students to explain concepts (why) not procedures (how). 3. There is no one right approach and mistakes are opportunities to learn. 4. Learn to listen. 5. Mathematical understandings will develop over time. 6. Help students express themselves more clearly. 7. Get students to talk to one another. 9. Confusion and struggle are a natural, necessary, and even desirable part of learning mathematics. 10. Create a learning environment where all students feel safe to share their mathematical ideas.	Observations of these Guidelines in the Role-Play

B. Have teachers choose a math task they already use with students and increase the level of rigor

1. In *5 Principles of the Modern Mathematics Classroom,* Aungst introduces the difference between an 'exercise' and a 'problem.' If we want to include more "problems" – that is, if we want students to develop a deeper understanding of math concepts -- we need to consider the *rigor* of the problems we assign. One useful tool to use is Norman Webb's Depth of Knowledge. Have teachers look at Webb's descriptions of the different levels of thinking along with the types of math problems that fit each of the first three levels.

Webb's Depth of Knowledge (DOK)	Example from Area and Perimeter	Example from Quadratics
Level 1: Recall and Reproduction (recalling basic facts)	Find the perimeter of a rectangle that measures 4 units by 8 units.	Find the roots of the equation: $y = 3(x-4)^2 - 3$
Level 2: Skills and Concepts (involves some decisions and skills such as comparing, organizing, and estimating)	List the measurements of 3 different rectangles that each has a perimeter of 20 units.	Create 3 equations for quadratics in vertex form which have roots 3 and 5, but have different max or min values.
Level 3: Strategic Thinking (involves planning, evidence and more abstract thinking – such as solving a non-routine problem or explaining the reasoning behind a Level 2 problem)	What is the greatest area you can make with a rectangle that has a perimeter of 24 units?	Create a quadratic equation using the template below with the largest maximum value using whole numbers 1 to 9 no more than once each: $Y = -\square(X - \square)^2 + \square$

2. Next have teachers choose a math task they currently use with students that fits into Level 1 above. Alone or in pairs, have teachers tweak their tasks to create a Level 2 or even a Level 3 problem they could use with their students.

C. Have teachers create norms for the first days back to school AND a rich mathematical task

1. Have teachers read Chapter 9 in *Mathematical Mindsets*. Discuss the chapter, then tweak the 7 norms introduced in this chapter (and outlined on the first page of this PD section (1. Everyone can learn math to the highest levels, etc.)) to fit each teacher's style. Have teachers think through how they want to introduce these during the first days of school. (Note that Boaler has 5 short mindset videos you can use with younger students at youcubed.org.)

2. Have teachers work together to take one of their math tasks and adjust it based on Boaler's 6 design principles in Chapter 5.

IV. ASSESS – What to Look For in Math Classrooms

Have your **leadership team** look at the chart with the overview of the three authors' philosophies at the beginning of this PD outline. Choose *five* areas your math teachers most need to strengthen. Then create a shared list of LOOK-FORS to use when instructional leaders observe math teachers. Brainstorm a list of LOOK-FORS that align with the *five* areas your leadership team chose. Below are a few suggestions:

5 Philosophical approaches we need to strengthen	What to look for when observing math instruction
Ex. There is no one right approach and mistakes are opportunities to learn. (Humphreys and Parker)	• Teachers explicitly tell students their brains grow when they make mistakes. • Teachers spend time with wrong answers rather than jumping to correct ones.
Ex. Chaos - Class is understandably messy when students are truly allowed to struggle with mathematical concepts (Aungst)	• Teachers provide open-ended questions that allow for messiness and failure. • Teachers give students plenty of time to experiment, and keep the stakes low • Teachers don't immediately rescue students who struggle, but allow for struggle.
Ex. Depth is much more important than speed. (Boaler)	• Teachers provide open-ended tasks that emphasize interesting pathways not speed.

