Ways of working with numeracy and mathematics leaders to improve student learning

Peter Sullivan
Monash University

The rationale and process for working with teachers

The following is a description of the rationale and focus of teacher learning sessions designed to support school numeracy and mathematics leaders in working with other teachers towards school improvement. Although it is recognised that teacher knowledge of mathematics is important for effective teaching, the program is focused on the pedagogical content knowledge of teachers or, in other words, the actions they take when teaching mathematics. The discussion and examples that follow are from a particular teacher learning initiative, but the principles and teacher tasks are relevant more broadly.

It was assumed that the participants in the sessions were committed to improving their own teaching but more importantly were willing to work with other teachers on this improvement. Being part of a broader regional initiative, the *Achievement Implementation Zones*, meant that the focus of the sessions went beyond offering options for the participating teachers and coaches, and had some of the aspects of a "train the trainer" approach.

The overall goals of the program, as they were presented to participating teachers, were to:

- improve the experience of students when learning mathematics;
- improve student numeracy outcomes, as measured by NAPLAN data;
- build a sustainable supported school based culture of teacher learning through talking about practice.

The foci of the respective sessions were to

- provide guidelines and illustrative exemplars of ways to enhance mathematics learning;
- pose directions for particular classroom investigations; and
- allow opportunities for teachers to share experiences.

The basic argument in working to improve mathematics learning in this way is that groups of teachers need:

- a rationale for change or improvement;
- sustainable and structured ways of working;
- ideas to try out that can form the basis of their discussion.

The following illustrates some of the particular experiences on which the participants in the workshops engaged that were intended to act as prompts for school based activities.

Ways of working

The fundamental approach is based on studying, elaborating and exemplifying some commonly agreed characteristics of effective teaching. There are many lists of characteristics of effective teaching, which are generally compiled theoretically, or from surveys, or from descriptions of exemplary teachers (see Clarke & Clarke, 2004; Hattie & Timperley, 2007; Education Queensland, 2010). Six key principles of effective teaching were extracted from these sets of advice and these form the basis of the sessions conducted with the school numeracy and mathematics leaders with the recommendation that these also be used as the basis of work with teachers.

The following presents these six principles of effective teaching of mathematics. Like many of the ideas or guidelines in this book, these recommendations are easier to list than to enact. The approach taken is to exemplify each aspect of these six principles so that the intent is clear, and to allow teachers to reflect on the challenges and opportunities in working with teachers on the individual principles.

It is stressed that these principles might seem obvious, and that they are already part of the practice af all teachers. Yet most of these are complex and subtle and require specific teacher actions to enact them in classrooms. The intent was to provide participants with opportunities for collaborative and reflective discussion about the general principles.

Principle 1: Identify big ideas that underpin the concepts you are seeking to teach, and communicate to students that these are the goals of the teaching including explaining how you hope they will learn

It goes without saying, regardless of the educational context, teachers are better able to support students when they know what they hope the students will learn. Hattie and Timperley (2007), for example, reviewed a large range of studies on the characteristics of effective classrooms. They found that feedback was one of the main influences on student achievement, and the key elements were that students received information on "where am I going?", "how am I going?", and "where am I going to next?" To advise the students interactively, it is important for teachers to know their goals.

An example of an activity within the workshops is the following task:

> Write some sentences that have 5 words, with an average of four letters per word (no 4 letter words).

Teachers are asked, after giving some possible responses, to discuss and report on the following prompts:

> What is the mathematical point of that task?
> What is the pedagogical point of that task?
> How do you make these points explicit to students?

The discussion has potential to reveal not only how critical it is that teachers answer these questions for every lesson they teach but also that the answers to the questions are far from obvious. Experience with working with teachers on such discussion prompts shows that they quickly improve their skills at responding. One participating teacher reported at a subsequent session that he had written the lesson focus on the board at the start of his year 10 mathematics classes for a series of lessons. In one lesson he neglected to write the goal, and the students insisted that he do this because they had found this action so helpful.

It is also stressed that it is important for teachers to explain explicitly the pedagogical point or how they hope the students will learn. For example, if the teacher is using group work with the intention that students discuss their strategies with each other, then this should be explained to the students. This is particularly important for those students who are less familiar with the goals and processes of schooling.

As with the other principles, it is argued that such prompts can form an important component of teacher planning session and in this way can contribute to sustainable improvement.

Principle 2: Build on what the students know, mathematically and experientially, including creating and connecting students with stories that both contextualise and establish a rationale for the learning.

This is central to effective mathematics teaching. In the teacher workshops, a range of examples are given that either show how particular concepts can build on student experience, or how story contexts can be used to connect the students with the task. The following is an example of a sequence of progressively more complex tasks that are presented to the teachers:

> Kevin drives 20km to work and back again. If the costs of running his car are $2 per kilometre, how much does it cost him to get to work and back again?
>
> Kevin drives 73km to work and back again. If the costs of running his car are $1.37 per kilometre, how much does it cost him to get to work and back again?
>
> Kevin drives 20km to work and back again. Julia, one of his work colleagues, lives on the way to his work, but 5 km closer. If the costs of running his car are $2 per kilometre, how much should Julia give Kevin to take her to work and back again?
>
> Kevin drives 73km to work and back again. Julia, one of his work colleagues, lives on the way to his work, but 13 km closer. If the costs of running his car are $1.37 per kilometre, how much should Julia give Kevin to take her to work and back again?

This is intended to illustrate a number of important pedagogical points:

a) that easy calculations and harder ones are done the same way, so if students are confronted by a difficult task it may be possible to simplify it using easier numbers;
b) that progressively increasing the complexity of the numbers and the context makes otherwise difficult tasks accessible;
c) it is possible to connect everyday contexts with meaningful mathematics.

It is also possible to continue this in interesting ways. For example, the task can be posed to extend the activity to algebra and even to using spreadsheets.

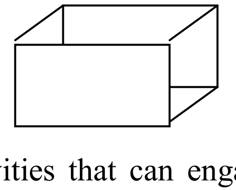
> Kevin drives x km to work and back again. Julia, one of his work colleagues, lives on the way to his work, but y km closer. If the costs of running his car are $ z per kilometre, how much should Julia give him to take her to work and back again?

Likewise, by extending the scenario a step further, as follows, it not only introduces the additional complexity but also leads to insights into the power of algebra which the equivalent task in posed in algebraic form.

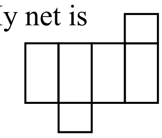
> Kevin drives 20 km to work and back again. Julia, one of his work colleagues, lives on the way to his work, but 5 km closer. Penny lives 10 km closer again. If the costs of running his car are $2 per kilometre, how much should Julia and Penny each give Kevin to take them to work and back again?

Also within this particular principle is the importance of building on what students know. While there are many ways that students can explore what students know, one underutilised resource is the analysis of NAPLAN items and results. To illustrate the way that this is done, the following was one item from the 2009 Year 5 NAPLAN test:

> The total number of faces, edges and vertices of this shape is 26. What is the total number of faces, edges and vertices of a square pyramid?
>
> 13 16 18 20



It is explained how this can be used as a prompt for particular activities that can engage students in thinking about the concepts involved. For example, the following is a set of statements that are put onto cards, with the intention that students match up those that refer to the same shape. For example, one set of these cards is as follows:

I have 12 edges	I have 8 faces and 8 vertices	I am a rectangular prism	My net is 

There are five such sets. The idea with these cards is that students sort them into groups, and in the process come to see them as different properties of the same object. The task is low in risk for students, and prompts communication about the learning. It can be easily extended to build a lesson around the key ideas of linking the language and properties of 3D objects.

Principle 3: Engage students by utilising a variety of rich and challenging tasks, that allow students opportunities to make decisions, and which use a variety of forms of representation.

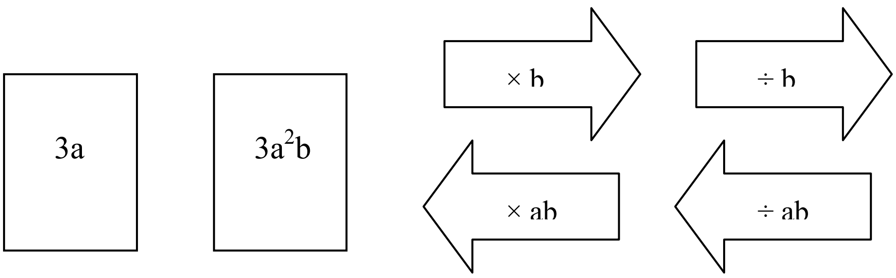
Sullivan, Clarke, and O'Shea (2010) sought insights into students' perceptions of the desired characteristics of mathematics lessons through their narrative responses. There was a range of interesting comments from the students, and they were extraordinarily articulate about the lesson features they value. The most striking aspect of their comments was their diversity, and there is clearly a range of ways in which students respond to lessons. There are two implications: one is that teachers need to use a range of lesson structures and student activities; and the other is that teaches should to describe to students the purposes of the different activity types.

Another second aspect of this principle is that students are more engaged when they make decisions. It is not proposed that students choose the content of the lesson, but students find it engaging to choose the ways of working, the type of examples, and the level of complexity.

A third aspect is the form of representation. While the overall goal with using various representations is that students link different perspectives on a concept, the different representations also allow connection with students who have particular preferences such as whether they are analytical or visual.

The "shapes" example above is challenging, it allows students to make decisions, and it prompts connecting different forms of representation.

The following is another example, which is a type of mathematical puzzle, adapted from a suggestion by Swan (no date). The Swan activity focuses on percentages, with fractions and decimal options. The puzzle involves a set of rectangular term (or number) cards and arrow operation cards, a subset of which could be:



In this case, the puzzle is to choose the two operation cards that can be placed between the two term cards to represent the connection. The point is that students have to look for the appropriate operation card to connect the terms and, by doing so, evaluate a range of possible operations simultaneously. It is also self correcting, in that there are unique operations connecting the terms. The tasks is challenging, the students make their own choices, and it provides a different way of conceptualising such relationships.

One aspect of the workshops was to focus on how teachers could create similar tasks for themselves for the particular skill on which they wish the students to focus.

The workshops also emphasise adaptability and flexibility in various ways. One is in the use of games.

There is a popular game called *Race to 10*. Students take turns at adding 1 or 2, starting at 0, with the winner being the person who says 10. There is a winning strategy which makes the mathematical point of the game more interesting. The point in introducing this game to teachers is that it can be played for many topics and many levels. For example, it could be:

> Race to 3, start at 0, adding ½ or ¼
>
> Race to 1, start at 0, adding 0.1, 0.05 or 0.15
>
> Race to 1000, start at 0, adding 50, 100 or 150
>
> Race to 0, start at 100, taking away any number from 1 to 12
>
> Race to 5x + 5y, start at 0, adding x, y or x + y

Each game can then be extended to more conventional exercises utilising the particular skill that has been practised.

Principle 4: Interact with students while they engage in the experiences, encourage students to interact with each other including asking and answering questions, and specifically planning to support students who need it, and challenge those who are ready.

Sullivan, Mousley, and Zevenbergen (2006) argued that one of the keys to successful mathematics teaching is to foster the sense in the class of participating in a mathematical community. Given that one of the principles above suggested that students be posed challenging tasks, this can create particular challenges, one of which is how the teacher can support students who experience difficulty with the challenging task.

Sullivan et al. (2004) argued that students are more likely to feel included in the work of the class, and to experience success, if teachers offer *enabling prompts* to allow those experiencing difficulty to engage in active experiences related to the initial goal task, rather than, for example, requiring such students to listen to additional explanations, or assuming that they will pursue goals substantially different from the rest of the class.

Of course, it may not be clear which aspects may be contributing to a particular student's difficulty. It is proposed that, by anticipating some of the factors, and preparing prompts that, for example,

- reduce the required number of steps, or
- simplify the modes of representing results, or
- make the task more concrete, or
- reduce the size of the numbers involved,

the teacher can explore ways to give the student access to the task without the students being directed towards a particular solution strategy for the original task.

In the workshops examples like the following are presented:

> 5 people went fishing. The mean number of fish caught was 4, and the median was 3. How many fish might each person have caught?

The task is challenging in that is offers a different way of conceptualising the concepts of the ways of describing the centre of the data. The teachers are invited to create enabling prompts for the task. This might involve, for students who are experiencing difficulty:

- suggesting they draw the people and the fish;
- advising they to ignore the median and focus on the mean;
- suggesting that you know someone caught one fish and someone else caught 2 fish; or
- revise the task to suggest there were only 3 people fishing.

Each of these suggestions has potential to reduce the demand of the initial task sufficiently that the students experiencing difficulty can complete the revised task and then proceed with the original task subsequently. They then remain part of the class community and can participate in any subsequent reviews.

There are also students who complete the task quickly. Such students also require some thought in that it is essential the the majority fo the students have time to complete the original task, yet these students also need a further challenge to keep them engaged. For these students it is proposed that teachers pose these students *extending prompts*.

One of the characteristics of open-ended tasks is that they create opportunities for extension of mathematical thinking, since students can explore a range of options as well as considering forms of generalised response. The challenge for teachers is to pose prompts that extend students' thinking in ways that do not make them feel that they are getting more of the same or being punished for completing the earlier work.

For the fishing task, for example, the teacher might ask students who finish quickly to:

- find a range of possible answers;
- consider the possibilities if the mode number of fish caught is 2;
- find the highest number of fish that an individual might have caught;
- create some similar problems for other students; and
- find a general way to describe the answer using words.

It is suggested that the process of creating enabling and extending prompts become part of the collaborative or individual planning routines in which teachers engage.

Principle 5: Adopt pedagogies that foster communication and mutual responsibilities by encouraging students to work in small groups, and using reporting to the class by students as a learning opportunity.

As with the others, this principle is deceptively simple and many teachers feel that they do these things already. Yet there are substantial advantages in teachers thinking about the specifics of what is possible.

There is a common lesson format that is recommended to Victorian teachers that in summary is described as: Launch; Explore; Summarise; Review. Yet this does not communicate the subtlety of the ways of working that are intended by this principle. One of the aspects of teaching that we can learn from the Japanese is the way they describe their lessons. As described by Inoue (2010), they use the following terms:

Hatsumon to mean the posing of the initial problem that will form the basis of the lesson, and the articulation to students of what it is intended that they learn.

Kikanjyuski the involves individual or group work on the problem. The intention is that all students have the opportunity to work individually so that when there is an opportunity to communicate with other students they have something to say. There is a related aspect to this described as *Kikanshido* which describes the teacher thoughtful walking around the desks giving feedback, and making observations that can inform subsequent phases in the lesson.

Nerige refers to carefully managed whole class discussion seeking the students' insights. There is an explicit expectation that students, when reporting on their work, communicate with other students.

Matome refers to the teacher summary of the key ideas

It is the latter two of these steps that can improve the learning experience of students. It is assumed that all students will have participated in common activities that could form the basis of common discussions and shared experience, both social and mathematical. These two aspects are reflected in Wood's (2002) emphasis on the interplay between children's developing cognition and the "unfolding structure that underlies mathematics" (p. 61) and "rich social interactions with others substantially contribut(ing) to children's opportunities for learning" (p. 61).

For the mathematical aspects, it is argued that students can benefit from either giving or listening to explanations of strategies or results, and that this can best be done along with the rest of the class with the teacher participating, especially facilitating and emphasising mathematical communication and justification. A key in such tasks such as this example is students having the opportunity to see the variability in responses (Watson & Sullivan, 2008). Cheeseman (2003) similarly argued:

> the critical issue is to think about drawing mathematics lessons to a close in the most effective and interesting manner. It is difficult to do so well and quite complicated because it involves much more than simply restating the mathematics. It encourages children to reflect on their learning and to explain or describe their strategic thinking. The end of the session give the opportunity for teaching after children have had some experience with mathematical concept. (p. 24)

The second aspect of reviews at the end of lessons is the contribution they make to social learning. The second aspect is related to a sense of belonging, but is also connected to building awareness of differences between students and acceptance of these differences. Such differences can be a product of the students' prior mathematical experiences, their familiarity with classroom processes (e.g., Delpit, 1988), social, cultural and linguistic backgrounds (e.g., Zevenbergen, 2000), the nature of their motivation (e.g., Middleton, 1995), persistence and efficacy (e.g., Dweck, 2000), and a range of other factors.

Interestingly, there is a structured process that the Japanese teachers use to build their capacity to do this, described as "lesson study" (Inoue, 2010). A simplified description of the process as it is presented to participating teachers is as follows:

> A group of teachers plans a lesson together
> One person teaches, the others watch and write reviews
> The lesson plan is revised after group discussion
> A different teacher teaches, others watch and write reviews
> This process cycles through

The major challenge in this for Australian teachers is having a second teacher in the room. To overcome some of the uncertainty in doing this, some specific actions are recommended for teachers.

For example, participants are advised that teachers and observers should recognise that:

- it is a learning experience for both, and the focus is awareness of action;
- ideally both the pedagogy and the content should be planned by both;
- there should be an agreed structure for feedback. The questions below might be a start;
- there should be a scheduled meeting both before and after the observation;
- there should be preliminary discussion about the language of review;
- the overall process of teacher learning, of which the observations are a part, should be clear.

It is recommended that the observed teacher:

- explain to the observer what you want them to do when in the classroom such as helping students, calling out suggestions, etc;
- identify a pedagogical focus on which you wish to receive feedback (also see the questions at the end of this for suggestions);

- invite the observer to say something to the class at the end.

It is recommended that the observer:

- Focus on the learning of the students, and be prepared to comment on anything unexpected (both good and bad)
- Make notes about the prompts below to inform your discussion

The following are also proposed as post- observation discussion prompts

What was the focus of the lesson?
What went better than you expected?
What could be done better another time?
Were the goals for learning made explicit?
Were the tasks set challenging for the students?
Were high expectations communicated?
Was mathematical understanding emphasised?
Did the tasks give the students opportunity to make decisions?
Was the lesson summary effective?
How was the diversity of readiness addressed?
What might you do next?

The emphasis in this required a commitment to, and the processes for engaging in, collaborate action to improve teaching and learning.

Principle 6: Fluency is important, and it can be developed in two ways: by short everyday practice of mental calculation or number manipulation; and by practice, reinforcement and prompting transfer of learnt skills

In the workshops there is limited time allocated to this principle in that it is the one with which teachers are most familiar. Many teachers have a range of strategies for developing fluency, and the notion of practice and prompting transfer is a common feature of commercially produced texts and worksheets. This principle does form part of comprehensive lesson planning nevertheless.

Conclusion

There are a number of issues that informed the structure and content of the teacher learning sessions. The six teaching principles gave a coherent theme to the program. It was intended that:

- the foci of the sessions be informed by research,

- the messages be focused and explicit;

- the processes suggested be both practical and sustainable;

- the recommended pedagogies be adaptable and transferable.

The strength of the program though was that it was part of a broader regional initiative that had the support of the Regional Director and staff, school principals, school leadership teams, and coaches.

References

Clarke, D. M., & Clarke, B. A. (2004). Mathematics teaching in Grades K–2: Painting a picture of challenging, supportive, and effective classrooms. In R. N. Rubenstein & G. W. Bright (Eds.), *Perspectives on the teaching of mathematics* (66th Yearbook of the National Council of Teachers of Mathematics, pp. 67-81). Reston, VA: NCTM.

Cheeseman, J. (2003). "Orchestrating the end" of mathematics lessons. In B. Clarke, A. Bishop, H. Forgasz & W. T. Seah (Eds.), *Making mathematicians* (pp.17-26). Brunswick: Mathematical Association of Victoria.

Delpit, L. (1988). The silenced dialogue: Power and pedagogy in educating other people's children. *Harvard Educational Review, 58*(3), 280–298.

Dweck, C. S. (2000). *Self theories: Their role in motivation, personality, and development*. Philadelphia: Psychology Press.

Education Queensland. (2010). *Productive pedagogies*. Downloaded in January 2010 from http://education.qld.gov.au/corporate/newbasics/html/pedagogies/pedagog.html

Good, T. L., Grouws, D. A., & Ebmeier, H. (1983).Active mathematics teaching. New York: Longmans.

Hattie, J., & Timperley, H. (2007) The power of feedback. *Review of Educational Research, 77*(1), 81-112.

Inoue, N. (2010, online). Zen and the art of neriage: Facilitating consensus building in mathematics inquiry lessons through lesson study. *Journal of Mathematics Teacher Education*. DOI: 10.1007/s10857-010-9150-z

Middleton, J. A. (1995). A study of intrinsic motivation in the mathematics classroom: A personal construct approach. *Journal for Research in Mathematics Education, 26*(3), 254–279.

Sullivan, P., Mousley, J., & Zevenbergen, R. (2006). Developing guidelines for teachers helping students experiencing difficulty in learning mathematics. In P. Grootenboer, R. Zevenbergen & M. Chinnappan (Eds.), *Identities, cultures and learning spaces (Proceedings of the 29th annual conference of the Mathematics Education Research Group of Australasia*. (pp. 496-503). Sydney: MERGA.

Sullivan, P., Clarke, D. M., & O'Shea, H. (2010, in press). Students' opinions about characteristics of their desired mathematics lessons. Paper presented at the conference of MERGA, July.

Swan, M. (no date). Using percent to increase quantities. Standards Unit trial materials. UK.

Watson, A, & Sullivan, P. (2008). Teachers learning about tasks and lessons. In D. Tirosh, D. & Wood, T. (Eds) *Tools and resources in mathematics teacher education* (pp. 109-135). Sense Publishers: Rotterdam.

Wood, T. (2002). What does it mean to teach mathematics differently? In B. Barton, K.C. Irwin, M. Pfannkuch & M. Thomas (Eds), *Mathematics Education in the South Pacific* (pp. 61-71), Mathematics Education Research Group of Australasia, Auckland, July.

Zevenbergen, R. (2000). "Cracking the code" of mathematics: School success as a function of linguistic, social and cultural background. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 201-223). New York: JAI/Ablex.