Introduction

What does it mean to problem solve? People of all ages engage in problem solving. A toddler might investigate how to obtain a toy that is just out of her reach. An adolescent might need to determine how to juggle multiple obligations, responsibilities, and desires effectively. Regardless of the task itself, people who engage in problem solving are seeking a solution to a challenging and novel task. They may bring a variety of knowledge to the process, investigate several possible solution strategies, and experience various degrees of success with their methods. They will experience failure, struggle, and triumphs, all of which contribute to their knowledge base for future problem-solving situations.

Problem Solving in Mathematics

These experiences also apply to problem solving in mathematical contexts. Problem solving in mathematics "means engaging in a task for which the solution method is not known in advance" (NCTM 2000, p. 52). This is an important definition to understand. If one is engaging in a routine task for which the solution strategy is already known, then it is not authentic problem solving. Instead, the routine task is an exercise, and the person engaging in the task is simply practicing a process or a skill or applying previous knowledge to a context (Eves 1963; Zietz 1999). Authentic problem solving, by contrast, means that the person must be engaged in developing new mathematical ideas or applying prior knowledge in new ways.

Although this will certainly involve failure, struggle, and successes, perhaps the most important of these is struggle. It is through active grappling with new concepts that we learn mathematics. Hiebert (2003) emphasizes that classrooms that promote students' understanding "allow mathematics to be problematic for students" (p. 54). As teachers, we are tempted to decrease our students' struggle by removing obstacles or showing them the way. However, it is important to realize that removing students' opportunities to struggle simultaneously reduces their opportunities to learn mathematics for understanding.

The Common Core State Standards for Mathematics (NGA Center and CCSSO 2010) has provided both an opportunity and a challenge for teachers to engage their students in mathematical problem solving. Beyond the clearly defined content standards, the Standards for Mathematical Practice (SMP) are a call for changes to classroom instruction such that students are engaging in challenging tasks, persevering through struggle, justifying and explaining their reasoning, and participating in critical and mathematically focused discourse that occurs throughout the classroom community. At all levels (K–12), students are expected to—

1. make sense of problems and persevere in solving them;
2. reason abstractly and quantitatively;
3. construct viable arguments and critique the reasoning of others;
4. model with mathematics;

5. use appropriate tools strategically;
6. attend to precision;
7. look for and make use of structure; and
8. look for and express regularity in repeated reasoning (NGA Center and CCSSO 2010, pp. 6–8).

Authentic problem solving provides opportunities for students to engage in these eight Standards for Mathematical Practice. Problem-solving tasks develop new understanding about particular content, but a student-centered implementation of the task—and some letting go on the part of the teacher—also allows students to develop mathematical processes or habits of mind that are identified by the practice standards. One might think of the content standards as *what* students are learning and the mathematical practices as *how* students are learning or engaging in the content.

Van de Walle (2003) discusses three characteristics of tasks that successfully promote student learning:

1. What is problematic must be the mathematics.
2. Tasks must be accessible to students.
3. Tasks must require justifications and explanations for answers or methods. (pp. 68–69)

Tasks that meet these descriptors present opportunities for students to develop new mathematical ideas; that is, the mathematics will be problematic for them, and there will be struggle. If students have already mastered the relevant content standards, then these tasks may be fun and engaging activities, but they will not be authentic problem-solving experiences.

Certain lesson formats are more conducive to creating problem-solving experiences that promote meaningful mathematics learning. Van de Walle, Karp, and Bay-Williams (2013) advocate for a three-phase lesson format. The three phases include the following:

1. *Getting Ready:* Activate prior knowledge, be sure the problem is understood, and establish clear expectations.
2. *Students Work:* Let go! Notice students' mathematical thinking, offer appropriate support, and provide worthwhile extensions.
3. *Class Discussion:* Promote a mathematical community of learners, listen actively without evaluation, summarize main ideas, and identify future problems. (p. 49)

The Launch/Explore/Summarize terminology used by the Connected Mathematics Project (Lappan et al. 2014) captures the same ideas articulated above and effectively illustrates the progression of participating in mathematics: Activating prior knowledge, engaging in mathematical thinking about a task, and extracting and summarizing the important mathematical ideas. Each of these phases is discussed in more detail in the following sections.

Launch

The Launch portion of a lesson is the teacher's opportunity to engage students in both the context and the mathematical ideas of a task. It is important to draw students into the circumstances of the task; This allows them to have a personal connection with the task, and helps them see how mathematics may be used in different ways in their or the lives of others. Engaging students in the mathematical ideas of the task is important as well, so students have a general understanding of the problem they are to solve.

There are multiple ways to engage students in the context of a task. Perhaps the most simple is to ask, "What do you know about … (e.g., Election Day)?" This kind of broad approach will provide a good sense of where the students are in relation to the context of the problem. It may surface misconceptions, but it may also bring out family traditions that can be shared so students' understanding and respect for others' experiences can be developed. Other questions that might be asked include the following:

- Has anyone here ever been …?
- How many of you like to …?
- How many of you celebrate …?

Sharing of students' beliefs and experiences provides the teacher an opening to add to the conversation, providing more information about the holiday or seasonal event to further connect students to the frame of reference.

Of course, depending on location, there may be very limited experience with the context of a task. If students have never seen snow before, it may be challenging to interest them in a situation about sledding or snowfall. If students have lived their entire lives in an urban setting, they may not bring prior experiences about camping or farming. If this is the case, it is still an excellent occasion to expand students' horizons. Perhaps some students have experience with these less common activities that they can share with the rest of the class. Online pictures and videos are good sources for context development.

Beyond clarifying the context of the task, the Launch is an ideal time to make sure that students understand the problem in which they are about to engage. To do this, students must employ the first Standard of Mathematical Practice, *Make sense of problems and persevere in solving them*. With teacher support, students should establish *what they know* about the problem. This may include knowledge gleaned from the problem itself, such as, "Avery has five friends," or inferences based on the information provided in the problem, such as "Six people will be making valentines." Students may be inclined to dive right into problems, performing operations on the numbers provided without thinking much about the problem itself before doing so. Asking them, "What do you know about this problem?" and listing their responses on a visual display requires them to think about the problem before jumping in.

Students should also be asked to determine *what they want to know*. Their initial focus might be on the answer to the problem. However, there may be questions that emerge as they make sense of the problem that they should be encouraged to recognize as important. Students' identification of these questions helps them realize that the problem solving will not be automatic; they may also be more metacognitively aware of some mental processes they are using while they work toward a solution.

The Launch is also an opportunity for students to develop a tentative plan for solving the problem. This can be tricky to negotiate; students sharing their plans can sometimes funnel other students' thinking at the cost of their own problem-solving strategies. Asking students to share their tentative plans with an elbow partner may alleviate this challenge. Voicing their plans may also help students identify places where their understanding of the task is still limited. Therefore, concluding the conversation with "Does anyone have questions about this problem?" provides a final occasion to clarify the context or the task before they set out to work.

Finally, it is important that students understand how the Explore portion of the lesson will progress. A variety of classroom materials should be made available for their use (e.g., rekenreks, Unifix cubes, color counters, etc.). Although some students may choose not to use manipulatives, these tools offer students who are reasoning less abstractly an entry point to the problems. Students should be assigned to partners or small groups, and be clear on the format expected for a final product.

It is important to note that the Launch portion of the task is not the place where the teacher does a similar problem with students or demonstrates how to solve the problem at hand. Doing either of these may drastically reduce the cognitive demand of the task, students' willingness to engage in the challenge, and the chance to learn important mathematics. While engaging students in preliminary processes for problem solving is necessary, both for tackling the task at hand and developing mathematical practices they can apply to any problem situation, this portion of the lesson should be limited to a meaningful ten minutes that effectively involves students in the context of the problem and the processes that will allow each student access to the problem itself.

Explore

As children engage in these tasks, the teacher's role is to provide appropriate scaffolding without removing students' opportunity to learn. How does a teacher do this? Ask questions. Listen carefully. Assess a child's understanding of the problem and determine where the more challenging aspects lie. Have children talk about their problem-solving strategies: what has worked and what has not.

The Explore portion of a lesson can be the most challenging for teachers who are not accustomed to teaching mathematics through problem solving. As teachers, it is our tendency to want to "help," make the path easier for our students and reduce their struggle. Although we do not want students to get to the point of unproductive frustration, we also need to be cautious about our "helpful" tendencies. Van de Walle's recommendation needs to be taken to heart for this section: "Let go!" (Van de Walle, Karp, and Bay-Williams 2013, p. 49).

Letting go as the students begin working on the task means that they need to be given several minutes to begin tackling the problem. For at least the first several minutes of the Explore portion of a lesson, these student groups or partners should be given the space to continue to process the task, share their initial plans, and begin exploring these plans.

Collaborative work will provide them more ideas for exploration and more insights about how certain mathematical ideas may apply to the task at hand. During this time, the teacher should circulate throughout the room with open ears, simply listening to the immediate challenges and insights. There may be common threads that surface across groups; however, allowing them the time to discuss without intervention may also provide the opportunity for them to resolve misunderstandings and differences.

As students get further in their problem solving, brief visits with each of the groups can keep them moving forward. Ask them questions about their reasoning, such as the following:

- Can you tell me why you decided to do this?
- What does this represent?
- What do you think your next step might be?
- What does this number mean in relation to the problem you're solving?

However, it is important to keep in mind when considering this section of the lesson that the teacher's task is to identify and understand students' mathematical thinking in relation to the task as well as their misconceptions and challenges. This is not possible without close and careful listening to students' discourse.

Task-specific questions or additional support may be necessary for groups that fail to find access into a task. A note of caution, however: This support should not be provided prematurely. Students learn through struggle, and as long as this is not unproductive frustration, they are probably grappling with important ideas and challenges.

Teachers may feel a similar temptation to rescue students who have gotten a wrong answer or are heading down an incorrect path. Again, this struggle is worthwhile, and as long as teachers are open and honest about honoring and respecting the learning that occurs through cognitive dissonance and/or mistakes, then children will see these as valuable learning opportunities as well. This may require a shift in what is honored and emphasized in the mathematics classroom. Processes, strategies, and the longitudinal development of mathematical concepts must be at the core, and mistakes must be valued as learning opportunities by both teachers and students.

Teachers may find it helpful to circulate with a clipboard throughout the Explore portion of the lesson so that misconceptions, challenges, insights, and strategies can be recorded. In many problem-solving lessons, a particular order for sharing strategies in the Summarize portion of the lesson is appropriate. Thus, knowing this order and being able to attend to the students who have reasoned in particular ways is worthwhile for leading students in a productive discussion of the important mathematical ideas related to each task.

Summarize

The Summarize portion of the lesson is the teacher's opportunity to engage the entire class in pulling together essential mathematical ideas. This needs to extend beyond a mere sharing of strategies. Although this is important, it is more important that students have a chance to discuss the mathematical ideas developed in each task, make connections between strategies, identify generalizations when appropriate to do so, and pose new problems.

Students should be exposed to the strategies that were used throughout the classroom community. There are multiple ways to arrange this exposure. Gallery walks, in which students circulate throughout the classroom to observe the work of others, is one effective way to share strategies. Sometimes, it may be appropriate for students to present their strategies to the whole class. At times, it may be important to provide time for everyone to share. Generally, however, we suggest that the choices for sharing be based on the mathematical ideas and strategies utilized by particular groups, with a long-range view of making sure that all students have opportunities to participate in this way. Although students may not have an opportunity in each lesson to present their thinking, they nonetheless ought to be engaged in discussing the strategies that are presented and in making connections between these strategies and their own.

Initially, students may not know how to engage in a community of learners that discusses important mathematical ideas. Although students may focus on nonmathematical ideas at first (e.g., “I like your drawing!”), the teacher can model appropriate probing questions and comments. For example, a teacher might offer comments similar to the following:

- “I'm interested in how you knew that you needed to add these numbers together. Can you explain that?”
- “I see that you used a lot of the same numbers in your problem-solving strategy, although you had a different way of solving the problem. Why do you think we are seeing the same numbers in these places?”
- “How do you think the first group's use of Unifix cubes is similar to your drawing?”
- “I'm not sure I understand what this picture represents. Could you explain that again?”

Students will learn from the teacher's modeling, but there should also be explicit attention to initiating a mathematical discussion. The teacher might ask students to think about the question she just asked and how it helped her clarify her own understanding about the mathematical ideas. The teacher might also consider providing sentence starters to help students structure appropriate and meaningful questions of their own.

It is critical that the mathematical ideas associated with problem-solving tasks be elicited and summarized during the discussion. Anticipating specific questions is helpful, but teachers should also experiment with questions that are particular to the strategies and misconceptions that surface in the classroom. Particular attention can be focused on generalizations that arise from students' thinking. What patterns do they notice? What do they expect would happen with a different set of numbers? What rules can be articulated, either informally or formally?

These generalizations may lead to opportunities for problem posing. Out of many good questions come more questions! Preparation to follow through with students' questions in subsequent problems or to record and post new problems encourages students to think about how these mathematical ideas extend beyond one problem-solving experience. Similarly, if extensions for students have been provided during the Explore phase, students should share the results of these extensions, making deliberate connections to the original task.

In this final discussion, mistakes and misconceptions should be tackled head on. The sharing of incorrect answers may discomfit teachers, but this should be an acceptable experience for students. This requires a safe community of learners in which students are comfortable with risk, expect mistakes to be made, and see opportunity for learning in these mistakes.

As students engage in more and more problem-solving experiences, they should be encouraged to take on more and more of the classroom discourse. Teachers should guide and facilitate rather than manage and direct. Students should be challenged to ask the questions and make the connections. Increasing the number of student comments occurring between teacher comments enables students to increasingly guide the discussion. The teacher needs to know the map, but oftentimes, the students are capable of choosing the route.

Of course, engaging in mathematical discourse like this takes time, effort, and patience. Primary children have plenty of ideas, but have difficulty articulating them. However, the only way to improve discourse is *through* discourse, so teachers should use rich tasks to take risks, and—as one preservice teacher described it—"embrace the train wreck" that may occur when following students' trains of thought. These "train wrecks" can lead to profound learning experiences!