CHAPTER 1

INTRODUCTION

Leading mathematical discussions can be both invigorating and challenging. It's easy to start a discussion by asking children to share their thinking. And nothing beats those moments when children proudly share something they figured out. But then what? Math discussions aren't just about show-and-tell: stand up, sit down, clap, clap, clap. Knowing what to do with students' ideas and teaching children how to meaningfully participate in discussions can be a lot more daunting.

We often worry during a discussion that students might get lost trying to keep up with what everyone is saying or that they may simply tune out when a lot of ideas are shared. And no teacher likes that uncomfortable feeling of putting a child on the spot. Yet when teachers plan for and steer productive discussions, children realize it's worth listening to one another and that it's okay to be pressed to say more.

Not all mathematical discussions have the same aim or should be led in the same way. In this book, we describe how considering your goals for math talk can help you better design discussions to meet those goals and teach children to participate meaningfully.

Our work with classroom discussions is guided by four principles:

1. Discussions should achieve a mathematical goal, and different types of goals require planning and leading discussions differently.
2. Students need to know what and how to share so their ideas are heard and are useful to others.
3. Teachers need to orient students to one another and the mathematical ideas so that every member of the class is involved in achieving the mathematical goal.
4. Teachers must communicate that all children are sense makers and that their ideas are valued.

These principles are at the heart of creating classrooms where children can participate equitably. Sarah Michaels, Mary Catherine O'Connor, and Megan Williams Hall (2010) write about classroom communities that live by these principles as engaging in "accountable talk," or "talk that seriously responds to and further develops what others in the group have said" (1). The challenge, of course, lies in putting these principles into action, which is what we hope to help you do.

The classroom vignettes in this book reflect insights we have gained into leading mathematical discussions while working closely with teachers who are committed to equity in students' achievements and learning opportunities. These vignettes are inspired by the way these teachers are creating practices to challenge school structures that sort and label children according to ability. We hope the everyday brilliance of the teachers and children in these pages will help us all strive toward creating classrooms that disrupt longstanding assumptions about who can and cannot excel in mathematics (Delpit 2012).

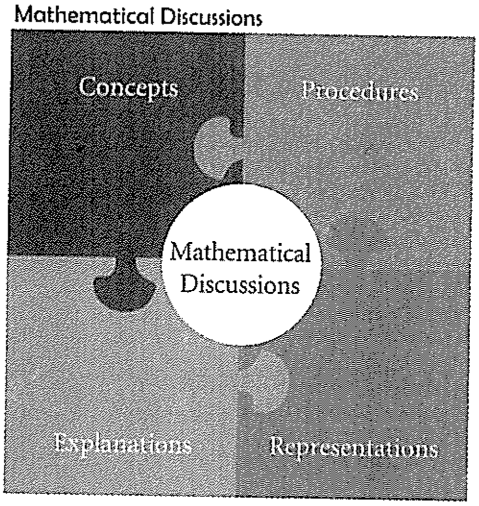
We'll now say a bit more about what we mean by each of these principles, give you a glimpse into the ways particular mathematical goals can help you run a discussion, and then outline what lies ahead in the rest of this book.

Principle 1: Discussions Should Achieve a Mathematical Goal

The mathematical goal acts as your compass as you navigate classroom talk. The goal helps you decide what to listen for, which ideas to pursue, and which to highlight. In *5 Practices for Orchestrating Productive Mathematics Discussions* (2011), Margaret Smith and Mary Kay Stein describe the importance of teachers clearly specifying the mathematical goal before planning out a discussion. The mathematical ideas at the heart of a lesson will help you distinguish between different types of classroom discussions you can have with your students.

Sometimes your aim is to have students share as many different ideas as possible in the discussion so they see a range of possibilities. We call this "open strategy sharing," because we're working on building students' repertoire of strategies. The class generates lots of ideas, and the discussion likely moves across a broad terrain that includes mathematical concepts, procedures, representations, and explanations (see Figure 1.1). Students listen for and contribute different ways to solve the same problem.

Figure 1.1 The Terrain in Mathematical Discussions



At other times you might want to focus the discussion on a particular idea. We call this "targeted discussion." Through targeted sharing the discussion zooms in on a particular idea. This more focused sharing involves specific goals, like defining and using key terms or concepts correctly, revising an incorrect strategy, or making sense of a particular representation. The students listen to and contribute ideas in order to move toward consensus. The table below lists the targeted discussion structures that will be the focus of Chapters 3 through 7.

Targeted Discussion Structure	Goal
Compare and Connect	to compare similarities and differences among strategies
Why? Let's Justify	to generate justifications for why a particular mathematical strategy works
What's Best and Why?	to determine a best (most efficient) solution strategy in a particular circumstance
Define and Clarify	to define and discuss appropriate ways to use mathematical models, tools, vocabulary, or notation
Troubleshoot and Revise	to reason through which strategy produces a correct solution or figure out where a strategy went awry

Principle 2: Students Need to Know What and How to Share

To make classroom discussions come alive, students need help knowing how to participate. When students have the chance to express their ideas, teachers have more information about what students *do* understand, what they are grappling with, and where they might be stumbling or confused. Students learn through classroom discussions *what* to share. In this book, we provide guidance for how to structure each type of discussion. We also pay attention to how students learn to contribute meaningfully in different types of discussions. We need to provide clear models for what an explanation sounds like and we need to judge, based on learning goals for students, what mathematical ideas require an explanation at any particular time. Students learn what to share as we prompt them to articulate important parts of their explanations. We can use and offer sentence starters that cue students to know what to say: "Explain to me what you meant by _______," "What would you do if the number was _______?" and "How is your way different from _______?" We also help students learn what to listen for so they can contribute to the conversation: "Listen for how she broke apart the numbers," "Think about whether you are understanding how she used the number line to show her thinking."

Similarly, students learn *how* to share through our explicit support. Reinforcing norms supports students in knowing how to share. For example, you might need to reinforce where to place oneself ("Stand here so we can see your work"); how loudly to speak ("Speak loudly so everyone can hear your idea"); and what tools to use ("Use the drawing in your journal to help"). We want to help children crack the code of being successful in meaningful mathematical learning at the same time that we tap into and draw on the resources they bring to the classroom (Aguirre, Mayfield-Ingram, and Martin 2013).

Principle 3: Teachers Need to Orient Students to One Another and the Mathematical Ideas

One of the challenges of leading discussions is bringing the whole class into the discussion. Most classrooms have several students who are eager to share and will always raise their hands and volunteer their thoughts. If we always call on these students, it's easy for others to remain passive or become anxious about how to enter the discussion. But that's not the only problem; if students have their hands raised just to get in their two cents, you'll end up with a bunch of ideas that don't build on each other or go anywhere. Teachers have to use strategies that help students learn *how* to attend to each other's ideas and the mathematics. We call this orienting students to one another and the mathematics. Teachers can draw attention to the meaningful contributions that all students make and can encourage students to take risks by "assigning competence," or identifying and naming students' specific contributions (Featherstone et al. 2011). The vignettes in this book will demonstrate many different ways that teachers advance the mathematical agenda of a discussion by strategically highlighting a student's insight or contribution, especially when a student might not be feeling confident about his or her standing with the rest of the class.

Principle 4: Teachers Must Communicate That All Students Are Sense Makers and That Their Ideas Are Valued

Our last principle is also the most important principle to put into action, because children have to be willing to take risks and put their ideas out there. Discussions obviously open up the possibilities that students will share their partial and incorrect understandings. How we respond to errors and partially developed ideas sends important messages about taking risks. It is not easy for students to express their ideas if there is a high burden to be correct and understand everything the first time around.

We need to remember that there's always a logic behind why students think the way they do, even if they seem off base. We also need to recognize publicly students' ideas, making sure we don't single out just a few students as mathematically "smart." There are many ways to be smart in mathematics, including making connections across ideas, representing problems, working with models, figuring out faulty solutions, finding patterns, making conjectures, persisting with challenging problems, working through errors, and searching for efficient solutions (Featherstone et al. 2011). Being smart in mathematics is not just about speed and accuracy. Vivian Paley writes that being curious about children's ideas signals to them that they are respected: "What are these ideas that I have that are *so* interesting to the teacher? I must be somebody with good ideas" (1986, 127). We want all students in the class to regard themselves as mathematical thinkers and to see themselves as people who can grow and be successful.

So how do these principles come together in teacher-led discussions? Let's consider two examples, an open strategy share and two types of targeted discussions, to see these principles in action.

Open Strategy Sharing: The Case of Mental Math

You might already have some experience leading discussions that fall into our open strategy sharing category. Discussing mental math strategies is a good example of open strategy sharing. It is a routine practice in elementary mathematics classrooms and is designed to build children's ability to flexibly, efficiently, and accurately compute. The teacher starts by posing a computational problem, such as $5 + 2$, $12 - 7$, 21×4, or $96 \div 6$, and invites children to share the different ways they figured out the answer.

Ms. Lind picks a multiplication problem for her fourth graders to solve as a warm-up to her main lesson. She's expecting to have the students spend about ten minutes sharing a few different ways of solving the problem. After writing 25×18 on the board, she steps to the side and provides time for her students to solve this problem mentally. As she sees that children have arrived at their solutions, she whispers to them to write their strategies in their math journals (she hopes this will help students remember the steps of their strategies). She circulates through the room, noticing the ways that students have approached the problem. When it looks like everyone has at least one solution, she asks the students to call out together what they got for the product. She records their ideas on the board to help make sure she doesn't put any one child on the spot to be correct or incorrect and to give herself the chance to see if there are multiple ideas in the room. She hears two different answers, 498 and 450. With all the ideas out, she begins by calling on a child who she could see has used a strategy that's fairly common in the class. Ms. Lind knows that asking Faduma to use her notebook will help her feel more comfortable sharing.

Ms. Lind: Okay, Faduma, tell us about what you wrote as you figured out this solution. I want everyone else to think about whether you are understanding what Faduma did and if you used a similar or different strategy.

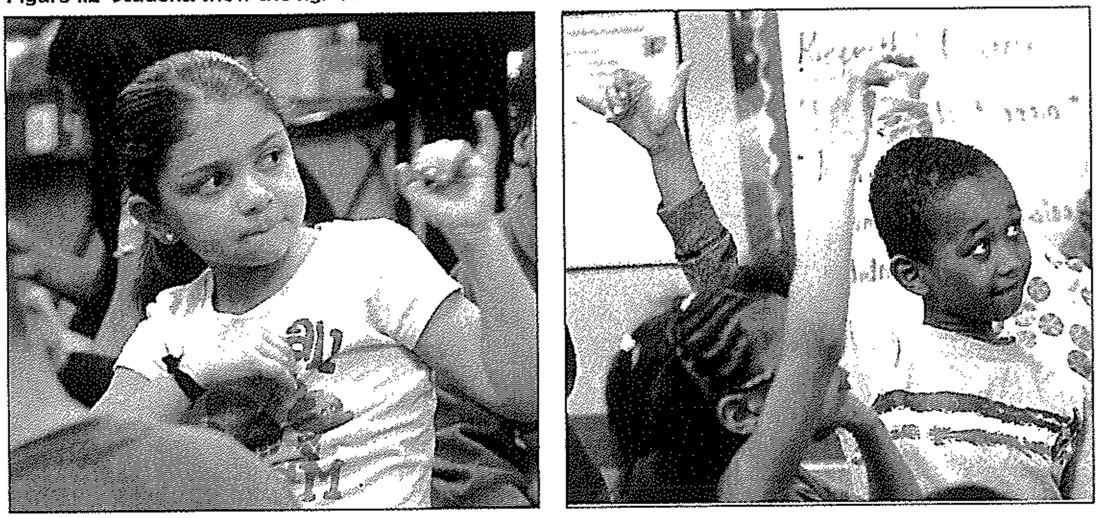
Ms. Lind's opening words help students know what to share and invite listeners into the discussion. She tries to help them know what to listen for.

Faduma: Since I can multiply numbers by 10, I broke up the 18 to a 10 and an 8. I multiplied 25 times 10 and 25 times 8. I got 250 plus 200, which is 450.

Ms. Lind: Thank you.

As Ms. Lind records Faduma's solution on the board, she notices that many students signal that they used the same strategy with the sign for "me too," inspired by the sign from American Sign Language. Children make the sign with one hand near their chest (or even close to their head), folding over their three middle fingers and rocking their hand back and forth (Parrish 2010; see Figure 1.2).

Figure 1.2 Students show the sign for "me too."



Ms. Lind: I've written on the board what I heard Faduma say. And many of you are showing me that you did the same thing. Who can add on to help us explain why we would split the eighteen the way Faduma did?

This question reinforces Ms. Lind's cue for listeners to see if they understand Faduma's ideas, which she gave at the beginning of the discussion.

Jordan: Well, it's like Faduma said, multiplying by 10 can be easier to do. So since one way of thinking about 25 times 18 is that you have 25 18 times, you can first do 25 10 times and then you have 8 more 25s.

Students signal agreement with Jordan. And Ms. Lind adds some words to what she has recorded to help make this explanation visible in the class display (see Figure 1.3).

Figure 1.3 Faduma's Strategy

25 × 18 = 450 (this means 25 18 times)

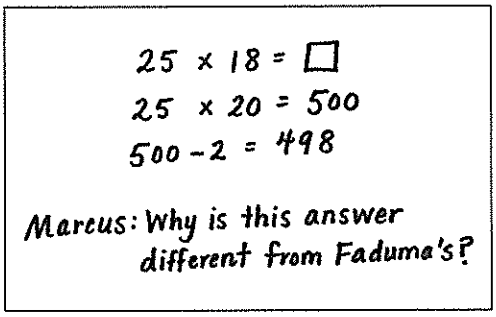
25 × 10 = 250 (this means 25 10 times)
25 × 8 = 200 (this means 25 8 times)

Ms. Lind: Does anyone have a question for Faduma?

Marcus: I do. I kind of solved it the same way but I got a different answer. Eighteen is close to 20 so I did 20 times 25 to get 500, but then I subtracted 2 to get to 498. I'm not sure why our answers are different.

Ms. Lind records Marcus's strategy on the board (Figure 1.4) to display for others what he is saying. She already begins to wonder whether she should pursue his question now or wait for another discussion.

Figure 1.4 Marcus's Strategy and Question



Ms. Lind: So you're really trying to make sense of Faduma's strategy through your own way. I'm writing your question up here, but before we take up your question, let's see if we can put one more strategy up here and maybe that will help us think about what is going on.

Ms. Lind makes this move given her goal of eliciting a range of ways to solve the problem. She also knows that Marcus's strategy, trying to round up and compensate for the difference, is not yet widespread in the class and may need some special attention.

Celia: *(Raising her hand to add to the discussion.)* I used what I know about quarters. Four quarters make one dollar. So 16 make 400 and then 2 more make 450.

Ms. Lind: *(Orienting the class to Celia.)* Celia, you gave us a lot to think about. What do you think Celia means when she says that quarters helped her solve the problem? And if you're not sure, you can ask her to repeat what she said.

Ms. Lind invites other students to be responsible for making sense of Celia's idea. She reinforces the idea that it's okay to ask Celia to share her idea again. Encouraging repetition gets several students to explain that Celia is thinking about the problem as 18 quarters, because quarters are worth 25 cents. Since 4 quarters make 1 dollar, Celia is thinking about 4 groups of 25 at a time. Ms. Lind puts Celia in the role of confirming or clarifying what her classmates say until they understand what Celia did. Ms. Lind writes Celia's strategy on the board (Figure 1.5).

Ms. Lind: *(Prompting students to think about the strategies shared so far.)* We seem to have three different strategies and two different answers. Could you turn and talk to your elbow partner about which strategies convince you and what questions you have?

Figure 1.5 Celia's Strategy

25 × 18 is like 18 quarters
Every 4 quarters is $1.00
So every 4 25s is 100
4 x 25 = 100
18 can be broken up into 4+4+4+4+2
100 + 100+100+ 100 + 50 = 450

This partner talk allows students to process what they have heard and gives Ms. Lind the chance to monitor the pairs and potentially select a few ideas to close out the discussion for the day.

Ms. Lind: I'm noticing as I listen to you that you are thinking about how your classmates broke up the numbers to multiply. Some of you are looking hard at Marcus's strategy and thinking that he changed the numbers. Marcus, can we come back to your strategy and spend some focused time on it? We can help you think through it more and see whether or not there's something we need to revise.

Ms. Lind ends this warm-up listening to the students' ideas and questions and tells them that in the next few days, they will address the questions that arose today. She's out of time to help the class think more about what went wrong in Marcus's strategy, so she assures them she will return to his strategy. This one problem, 25 × 18, generated many different ideas (which was Ms. Lind's goal for the discussion) and, as this short excerpt demonstrates, took the class into a broad mathematical terrain of interrelated concepts, procedures, representations, and explanations. Using the structure of open strategy sharing allowed the class to express and draw upon their ideas but not to linger extensively on any one idea. To spend more time on individual ideas, Ms. Lind needs to plan for targeted discussion.

Targeted Sharing: Two Follow-Ups to Mental Math

The open strategy sharing allows Ms. Lind to size up what ideas she needs to work on further with her students and to plan for a targeted discussion. She makes these decisions in the context of her unit and grade-level goals. For example, her students might benefit from dissecting a compensation strategy (i.e., rounding one of the numbers and adjusting the product, as Marcus attempted to do) or developing their skilled use of arrays to produce a representation about why Faduma or Celia's strategies worked. Ms. Lind wants her students to ground their use and justification of numerical strategies in both array and grouping models. She also wants them to learn to contextualize their strategies in story problems. These are two ideas emphasized in the Common Core State Standards for Mathematical Practice (Common Core 2012). She recognizes that she cannot meet all of her goals in one discussion, and her students could benefit from a focused discussion on using models and creating story problems. These observations lead Ms. Lind to plan for targeted discussions, which bring to the foreground particular concepts, procedures, representations, and explanations.

To illustrate a bit more deeply what we mean by targeted discussion, we offer two brief examples. Ms. Lind could use the targeted discussion structures Why? Let's Justify and Troubleshoot and Revise to highlight important mathematics that emerged from the open strategy sharing discussion. Please note that in the first example, we use an array model for multiplication, and in the second example, we purposefully switch to using a grouping model in order to show the possible choices that Ms. Lind could make. You'll be able to read more about Why? Let's Justify and Troubleshoot and Revise in Chapters 4 and 7.

Example 1: Why? Let's Justify

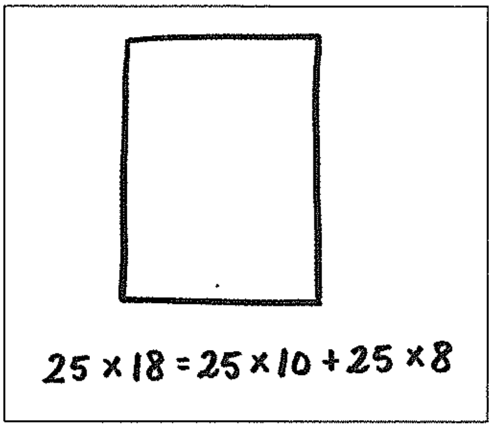
Connecting numerical strategies to a visual model is one way of making sense of why a strategy works; the model serves as a resource for children to verify their attempts at breaking apart a problem into smaller chunks. The goal of the Why? Let's Justify discussion structure is to figure out why a particular mathematical strategy works. Let's drop in on Ms. Lind's class as she leads a discussion to go further with the class in explaining the steps Faduma took.

Ms. Lind: Yesterday as we were listening to people solve 25 times 18, I realized it has been a while since we worked with arrays. I thought an array would be useful to explain what is happening when we break apart numbers to make a problem easier and how to make sure we've accounted for 18 groups of 25.

Ms. Lind puts an array on the board with Faduma's solution beneath it (see Figure 1.6) and asks students to draw, mark up, and label the array in their journals so that it matches this numerical strategy.

By walking around the room as students are working with the array in their journals, Ms. Lind can make purposeful choices about which students she will invite to share. She lingers over the shoulder of Celeste, who is dividing up the 18 into 10s and 1s, and thinks this idea will provide good fodder for agreement and possibly disagreement about how the array can match Faduma's solution. She is interested in inviting Celeste to share not only for her interesting ideas but also because Celeste tends to be quieter during discussions, and Ms. Lind is working to help her see herself as someone with good ideas. She kneels down next to Celeste and asks her if she'd be willing to share her drawing of the array with her classmates. Celeste nods, and Ms. Lind calls the group back together.

Figure 1.6 Ms. Lind draws an open array to show Faduma's strategy.

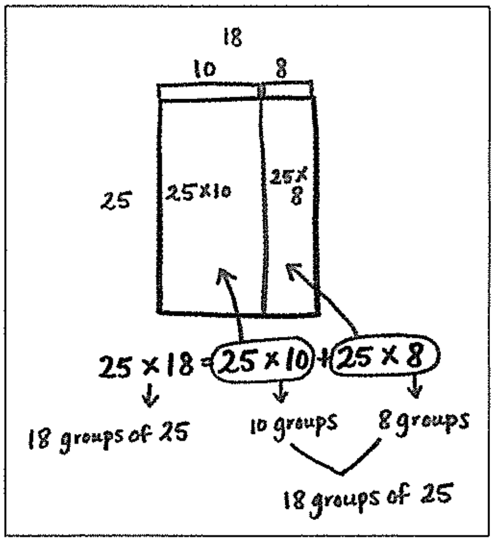


Ms. Lind: Celeste has an idea about the array to offer us. Please look up to the screen at this drawing of the array for 25 times 18. I want you to see if you can make sense of how she divided up the array.

Celeste shares her drawing, explaining how she thought about breaking up the 18 into 10 and 8. As students engage in whole-group and partner sharing about the array, the discussion evolves, with students adding on to prior contributions until a full annotation of the array is shared and it corresponds to the numerical recording of Faduma's strategy (see Figure 1.7).

Ms. Lind's goal is to get to a place where students can see that Faduma's approach began

Figure 1.7 The annotated array of Faduma's strategy looks like this by the end of the discussion.



with figuring out 10 groups of 25 and then adding on 8 more groups of 25 to end up with 18 groups of 25 altogether. This targeted sharing asks the class to focus in on one solution and explicitly map the connections between the symbolic and visual representation. In Chapter 4 we dig more deeply into how the teacher navigates these discussions to support student sharing and orient students to one another and the mathematics.

Example 2: Troubleshoot and Revise

It can be quite powerful for a classroom community when students share ideas that aren't quite right yet and seek the help of their classmates. A student seeking peer feedback is valued as having a good kernel of an idea that needs to be developed, and his or her classmates can be motivated to work through the issue. Ms. Lind could use the Troubleshoot and Revise discussion structure to help Marcus and his classmates make sense of where Marcus's strategy went astray and how to revise it to make it work. This lesson could take place the same or next day. After being asked whether he felt comfortable conferring with his classmates to find an answer to his question, Marcus willingly recaps his strategy aloud.

Marcus: I did 20 times 25 to get 500 and then I subtracted 2 to get 498. I kind of think it should work because 20 is just two more than 18. But I'm not sure why I'm not getting the same answer as Faduma. I think I should be.

Ms. Lind could ask students to use an array model to help students troubleshoot and revise Marcus's strategy, and if this discussion actually followed on the heels of the first vignette in this chapter, it would be appropriate for Ms. Lind to use the same model. However, to broaden the representations we use and to provide an example of how contextualizing the numbers and operation in a situation can also be helpful with the revision process, we are going to use a grouping model in this next vignette.

Ms. Lind: Marcus had a great way of beginning this problem. By changing the 18 to 20, he started by making the problem easier for himself. It might help us to put these numbers into a story. Let's imagine Marcus had 25 packs of colored pencils, with 18 pencils in each pack.

As she says this, Ms. Lind values Marcus's idea that he needed to make an adjustment when he rounded one of the factors to 20. Often students are *not completely wrong, and we can highlight their good thinking. She is also intentionally selecting a familiar problem context to make sense of the changes to the numbers.*

Andre: Oh, I see. When Marcus changed the numbers to 20 times 25, it made it like there were 25 packs of pencils with 20 pencils in each pack. But, we need to have 18 pencils in each pack, not 20!

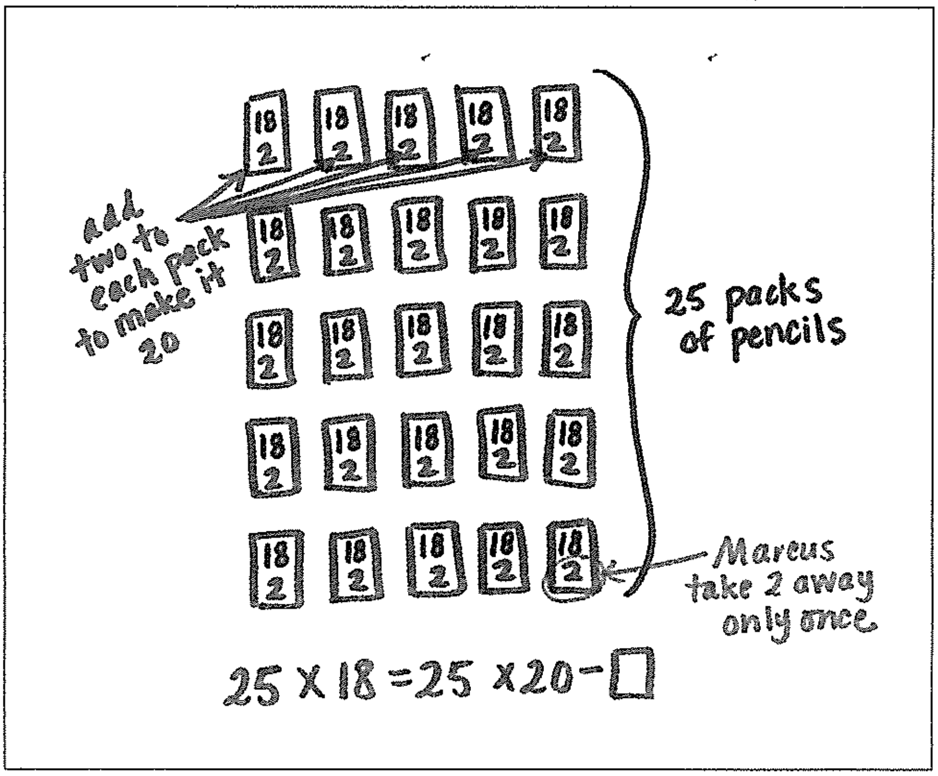
Ms. Lind: Okay, let's draw the 25 packs of pencils with 20 pencils in each pack.

Drawing these new pencil packs can help the class keep in mind what happened as Marcus's numbers changed from 18 to 20.

Ms. Lind: So, what needs to be removed from each pack to go back to having 18 pencils in a pack instead of 20?

She uses partner talk to engage all the students in considering how to take two pencils out of each pack and how to change the drawing to make it match 25 × 18 (see Figure 1.8). Ms. Lind circulates among the students as

Figure 1.8 These notes on the board supported Marcus in revising his strategy.



partners discuss this problem to select who should alter the drawing on the board. The drawing can be annotated to show that 50 pencils altogether need to be removed from the product. This will clarify what happened when Marcus subtracted 2 from only 1 of the packs instead of removing 2 from all 25 packs.

Ms. Lind ends the discussion by asking the class to revise Marcus's strategy.

Ms. Lind: Okay, we have worked together to figure out why Marcus's answer was different. Who can say again why his answer was different?

Together the class concludes that when Marcus changed the problem to 25 × 20, he needed to subtract 25 groups of 2 to make sure each pack only had 18 pencils.

Ms. Lind: *(Summarizing the group's thinking as she writes on the board.)* So it looks like we agree that the equation that shows Marcus's strategy should read 25 times 18 is the same as 25 times 20 minus 50.

She hands out an exit card posing a new problem: "How would 15 times 20 need to be adjusted in order to solve 15 times 19?" As a formative assessment strategy, the exit cards help her see how this discussion helped students think about using a compensation strategy. Students fill out these exit cards at the close of a lesson. Ms. Lind will review them in order to assess what students are learning and what they might still be grappling with. What she learns from the exit cards will help her plan for subsequent lessons. You can read more about Troubleshoot and Revise in Chapter 7.

Looking Ahead

We believe that the way teachers and students talk with one another in the classroom is critical to what students learn about mathematics and how they come to see themselves as mathematical thinkers. In our classrooms students should feel that they belong and that they can be successful. Talk is an important way to build that sense of community and to help children grapple with important mathematical ideas. Group discussions can energize children if we are careful about how we teach children to listen to, respond to, and engage with one another's ideas. The time we invest in helping our students learn to participate productively in discussions can result in a huge payoff.

We hope the short vignettes in this chapter begin to help you see our principles at work and the differences between open strategy sharing and targeted discussions. In open strategy sharing, Ms. Lind and her students came up with several ways of thinking about 25 × 18. Targeted discussion helped Ms. Lind zoom in on a few key ideas that came up in the open strategy share. In the rest of this book, we dig more deeply into these structures.

Chapter 2 describes open strategy sharing in more depth. We discuss situations in which teachers might choose an open strategy share and how teachers can lead the discussion to help students listen and contribute to the discussion without getting bored or lost.

We begin our discussion of targeted discussions in Chapter 3 with a structure that naturally extends open strategy sharing, Compare and Connect. The important difference between open strategy sharing and Compare and Connect is that in Compare and Connect the teacher not only elicits strategies but also asks students to find the mathematical similarities and/or differences among them.

We want students to develop a repertoire of strategies, but we also want them to be able to explain why those strategies work. Chapter 4 takes us back to the Why? Let's Justify structure. The goal is to generate justifications for why a mathematical strategy makes sense. This type of discussion typically focuses on just one kind of strategy or procedure. The students are all oriented toward producing a viable explanation. This chapter will help you understand the difference between describing the steps in a strategy and justifying them.

While it is possible to solve some problems in many different ways, students also need opportunities to become more selective about when to use a particular strategy. Chapter 5 takes on this issue by describing a structure we call What's Best and Why? In this discussion structure, the teacher begins not by eliciting ways to solve a particular problem but by (1) showcasing a particular strategy and then asking students to generate an effective use of that strategy or (2) showing a few different ways to solve a problem and asking students to figure out which is the most efficient strategy for this problem.

Teachers often introduce new mathematical models (e.g., number line or array), tools (e.g., a tens frame, the hundreds chart), vocabulary, or notations into mathematical discussions. Models, tools, vocabulary, and notations are all considered mathematical objects. Chapter 6 will highlight how those objects could be the focus of a discussion structure we call Define and Clarify. We consider when such discussions could occur (e.g., when models, tools, terms, and notations are first being introduced or when students have had a chance to use, say, a certain model, but the teacher wants to refine its use). The teacher orchestrates these discussions by modeling the use of the new model, tool, or idea and helps students determine incorrect versus correct usage of that mathematical object.

Chapter 7 more closely deals with how teachers can use errors as opportunities for advancing mathematical thinking through Troubleshoot and Revise, a discussion structure you've already glimpsed. This chapter showcases teachers prompting students to reconcile different strategies in order to defend the correctness of one of the solutions or to engage in a conversation with classmates to find where missteps occurred in a problem-solving attempt and what revisions are needed.

We end the book by summarizing in Chapter 8 the big ideas we've shared and by providing guidance about how to choose goals for discussion and make productive use of the discussion structures within your own curriculum.

To support your teaching, we've included a set of planning templates for the various discussion structures (Appendixes A-F). We've also provided several lesson protocols from the routine instructional activities that appear throughout the book (Appendixes G-I). And, finally, you will also find a list of books and videos available on the web that can help you envision some of the practices and moves you'll see described in the vignettes (Appendix J).