4 Communication

> To effectively communicate, we must realize that we are all different in the way we perceive the world and use this understanding as a guide to our communication with others.
>
> —Tony Robbins, Motivational Speaker and Life Coach

> If you can't explain it simply, you don't understand it well enough.
>
> —Albert Einstein, Theoretical Physicist, Authority on Philosophy of Science

THE MYSTERIOUS MATHEMAGICAL MIND TRICK

"Go right to your seats and get ready for math," I said as I brought my fifth graders back to the classroom from lunch.

"What's that, Mr. Aungst?" asked Kelly, pointing at a large manila envelope propped on the chalk tray. Several students paused to look. Momentarily confused, I turned and saw a mysterious and somewhat ominous directive written in large, red, block letters: "DO NOT OPEN YET."

I scratched my head. "I'm not sure. I didn't notice that before. It wasn't there when I left to pick you up from the cafeteria. Very odd." I started to move toward it, then hesitated. "No, I suppose we should follow the directions and not open it yet."

"When will we open it?" Kelly asked.

"I imagine we'll know when it's the right time."

A few minutes later, everyone had settled into their seats. "You know, this mysterious envelope has me thinking," I said. "Who likes magic tricks?" Lots of hands went up. "Great! How many of you believe that I can read minds?"

There were a few chuckles around the room, and a couple of skeptical looks, but no hands this time.

"I really can. It's a necessary skill for teachers. Would you like me to prove it?"

I could sense a few students were intrigued. A few said, "Yes!"

"OK, then. Everyone get out a clean sheet of paper and a pencil. I'm going to give you a series of instructions. These steps are designed to clear your thoughts so that it will be easier for me to read your mind." I began to give them instructions. In order for you to experience it from the student's perspective, follow along with this adaptation of the classroom activity. At each step, start with the answer from the previous row. Instead of using a classroom math textbook for the last step, use this book:

First, write down any two-digit positive integer.	
Multiply the number from the first row by nine.	
Add the digits of the product. If the sum has two digits, add them again. Call this sum "W."	W:
Multiply W by 13. Call this product "P."	P:
Turn to page number P in *5 Principles of the Modern Mathematics Classroom*. Ignoring any headings, begin on the first line of the main text on that page and count to word W. Write this word here.	

When doing this lesson with students, I'd typically have them use their math text and choose any picture from page P. Let's return to the classroom.

"Has everybody chosen their picture? Great! Now, focus on that picture so I can read your minds." I proceeded to perform a ridiculously elaborate ritual, telepathically gathering all of their pictures, selecting one, and imprinting it from across the room onto whatever was inside the sealed envelope.

Throughout the activity, I had not touched or even approached the envelope. "Kelly, you found the envelope. Can you go open it for us?"

Before I reveal the end of the story, I want you to return to the word you found in this book. Now turn to the next page to Figure 4.1. There you will find a group of words arranged in a grid. Your word is the one at the exact center of the grid, the only one in bold. Am I right? Did I read your mind?

Returning to our classroom, Kelly opened the sealed envelope, revealing a single sheet of paper with a picture on it. "How many of you were thinking of this picture?" Every hand was raised. There were a few shocked faces, but most of the class was now curious.

"See, I told you I could read minds. How many of you believe me now?" Kelly was not raising her hand. "Why don't you believe me, Kelly?"

"How could everyone get the same picture?"

"That's a really good question. Maybe you all started with the same number?" A quick check proved that theory wrong. And yet everyone ended up on the same page. Just like you located the ninth word on page 117 of the book in your hand.

So did I really read their minds? And yours? Of course not. I did say at the beginning it was a trick.

"And that's my point," I told my students. "Teachers cannot read minds. And yet so much of our job involves understanding what you are thinking. So, if I cannot read your mind, what's the only possible way for me to know what you're thinking?"

"If we tell you?" Kelly responded.

That is why Communication is the second Principle of the modern mathematics classroom.

FIRST AND FOREMOST, COMMUNICATE

We have established that math is not primarily about computation or a checklist of content and skills. It is about solving problems, and problems are solved through communication.

In a classroom based on the 5 Principles, students must learn to communicate frequently about problems and how they solve them. They focus on vocabulary, writing, and the formulation and support of mathematical arguments.

Therefore, a math classroom should first and foremost be a communication classroom, and should be filled mostly with talking and writing. Most math learning takes place when students are talking to each other and writing—in English, not just math symbols. Students should explain, argue, defend, critique, and discuss ideas.

Most math learning takes place when students are talking to each other and writing—in English, not just math symbols.

Figure 4.1 Words for Use in the Mysterious Mathemagical Mind Trick

swift	culture	some
the	**also**	below
math	guess	program

Students should write on a daily basis, produce longer math pieces weekly, and share their writing with multiple audiences.

We are going to borrow a lot in this chapter from language arts instruction, including best practices for English Language Learners, because teaching communication in math is very similar to teaching communication in English. The context and content may be different, but the goals are the same: to successfully convey ideas between two people.

SIX LEVELS OF MASTERY

Research supports the premise that communicating about math is essential to full understanding. David Sousa (2008) describes six levels of mastery that a student must move through in order to learn and retain mathematical concepts (see Figure 4.2 for an example illustrating the six levels). Pay attention to how much of math mastery is dependent on communication:

- Level One: Connects new knowledge to existing knowledge and experiences
- Level Two: Searches for concrete material to construct a model or show a manifestation of the concept

Figure 4.2 Illustration of Sousa's Six Levels of Mastery

Level One: The student recognizes that fractions are related to division, and that $\frac{3}{4}$ is the same as $3 \div 4$.

Level Two: The student measures out three cups of sand into a container, then divides the sand into four equal amounts and measures each pile to see that it contains $\frac{3}{4}$ of a cup.

Level Three: The student draws a picture showing that $3.00 can be divided into four equal amounts by exchanging dollars for quarters and rearranging them.

Level Four: She writes $\frac{3}{4} = 3 \div 4$ and $\$3 \div 4 = \$0.75$

Level Five: The student then solves the following problem, "Elana's band teacher is holding a three-hour dress rehearsal right before the winter concert. The band has four songs to practice, and the teacher wants to spend the same amount of time on each one. How long will the band practice each song?"

Level Six: The student explains orally how fractions and division are related to a peer or to the class, or explains the reasoning behind her solution to the band practice problem.

- Level Three: Illustrates the concept by drawing a diagram to connect the concrete example to a symbolic picture or representation
- Level Four: Translates the concept into mathematical notation using number symbols, operational signs, formulas, and equations
- Level Five: Applies the concept correctly to real-world situations, projects, and story problems
- Level Six: Can teach the concept successfully to others, or can communicate it on a test

(Sousa, 2008, pp. 169–170)

GRADES K–3

Watch Your Language!

MP6

Several years ago, I had the opportunity to visit Madrid. It was the first time I had been in a foreign country where the local residents didn't speak any English. My own knowledge of Spanish is limited to counting to ten, saying thanks and good day, and a few colors and food items. (Thank you, *Sesame Street*.)

It was an extremely enlightening experience. I knew for the first time what it was like to need to communicate, but not be able to express myself. It was frustrating for me, and for many of the *madrileños* with whom I interacted.

> As adults, we fluently read the symbolic language of basic math and translate it automatically into its English equivalents. It's easy to forget that for our students, they are still two different languages.

For young children, learning math is like learning a second language. Complicating things, we use English words for much of the work, but we also mix in an entirely different symbol set with meanings of its own. As adults, we fluently read the symbolic language of basic math and translate it automatically into its English equivalents. When we speak math aloud, we use English words. It's easy to forget that for our students, they are still two different languages.

Our colloquial use of some words is also different than their more precise use in math. Further complicating things, math has specific academic language for particular situations that we speak about more broadly in common conversation. Let's take the word "answer." Beyond its meaning as the response to a question, we also use it to mean the result of a mathematical computation, or the solution to a problem. However, mathematics has more precise terms that students need to begin using as soon as they begin learning math. Use the word *solution* when talking about problems. For computations, teach the terms *sum*, *difference*, *product*, and *quotient*, and help students understand and ingrain their meanings.

Teach academic math vocabulary the same way that you would in a language arts class. To accomplish this, Marzano and Pickering (2005) provide a straightforward six step process:

1. Explain the new word. This must go beyond reciting or copying a definition. Use examples, imagery, and nonlinguistic representations.

2. Students now restate in their own words. Repeat this word both verbally and in writing, as appropriate for the developmental level of the child.

3. Students should now create their own nonlinguistic representation. This can be a picture, a model, a demonstration, or their own symbolic representation. Some students with lots of background knowledge may give you the standard symbols; acknowledge this, then encourage them to come up with their own representation so that they can show what they know about the word.

4. Regularly engage students in activities that allow them to deepen their understanding of the words, such as categorizing, sorting, semantic mapping, analyzing root words and affixes, or creating analogies and metaphors.

5. Students should also have discussions about the words with partners and small groups. They can explain their understanding, their visual representations, or new ideas they have learned about the meaning of the word. This can and should happen by itself and also in connection with the activities in Steps 4 and 6.

6. Play games to explore and review the vocabulary terms. This could also happen as part of games played to review and understand other math concepts, but vocabulary should also receive its own dedicated review time.

We also need to explicitly teach the symbolics of math as its own language for communicating mathematical ideas in short, simple ways. Talk about how you are really translating between two languages when you discuss something in English and then show it in mathematical symbols.

This became clear to me several years ago during a lesson on Egyptian hieroglyphs with my fourth grade gifted class. The activity took place immediately after I taught a fifth grade math lesson on equations with variables. While explaining to my fourth graders that the Egyptians often wrote hieroglyphs out of order, and that some of the symbols represented sounds, some ideas, some were modifiers, and some had different meanings depending on the context, I realized that this is exactly how our mathematical symbol system works.

One of the frustrations that I have when teaching math is that students tend to read from left to right, and often when they get to something that hangs them up, they just stop there and don't try to figure it out. Beyond the most elementary number sentences (2 + 3 = 5), this approach doesn't work well. It is essential for students to learn that sometimes you read from right to left, sometimes you read from the middle out, and sometimes you have to piece different parts together until the whole equation makes sense.

It's routine in elementary math instruction to ask students to write numbers in multiple forms so that they can see the relationships between them. For example:

Standard form: 728

Expanded form, explicitly representing the place values: 700 + 20 + 8

Word form, showing how we speak and read the number: seven hundred twenty-eight.

But, it's rare that we do the same for any of the larger symbolic representations of math, like expressions and equations. Make time in your math instruction to try something like this:

Standard form: 11 + 24 = 35

Word forms:

- eleven plus twenty-four is equal to thirty-five
- the sum of eleven and twenty-four is thirty-five
- thirty-five is equal to the sum of eleven and twenty-four
- when you combine the quantities eleven and twenty-four, the total is thirty-five

See how there are several ways to express the same symbolic statement in English, and the English statements don't necessarily correspond to the symbols in left-to-right order? Students need to see that all of the word forms are versions of the same

Ask students to practice translating their words into mathematical symbols and vice versa.

mathematical concept, expressed by one symbolic statement. Ask students to practice translating their words into mathematical symbols and vice versa.

Digital Tools and Resources for Communication in K–3

Although it is important to get kids writing about math as early and often as possible, the focus during these years should be on building verbal fluency with mathematics. This emphasis leads to much better written work in the upper elementary grades.

Any means of digitally recording students is a terrific way to help them improve these skills. Recordings can be used for math instruction in a number of ways:

- Make a video of students talking about their math work, then have them watch their own explanations to reflect on what makes sense and how they can explain things more clearly.
- Keep an archive of the best examples of student math talk and use them as demonstrations in addition to your own modeling and think aloud strategies.
- Create a class podcast or video blog to share student math work with parents. This creates a different audience for the communication, since parents weren't in the room during the lesson and will need a different kind of explanation to understand what is going on. This also opens up the possibility of two-way communication about math.
- Have students record their own instructional videos about vocabulary for either younger students just learning the terms or as review for students in the same grade.

GRADES 2–5

Conversations Are More Important Than Computations

MP2

In the middle of a demonstration activity I was doing in a sixth grade classroom, the classroom teacher pulled me aside. "I see what you're doing here and I like the idea," she said. The students were working in small groups to solve a small, but challenging problem: *with a small box and a large sheet of paper, figure out how to completely cover the box with one piece of paper so that there are no gaps or overlaps.* There was a lot of activity, a lot of mess, and most importantly, a lot of student talk. Her students were talking animatedly about the problem. "I understand this," she continued, "but I get very uncomfortable when students are out of their seats, moving around, and talking."

The math concept behind this activity is understanding that a prism, a three-dimensional figure, can be represented by a *net* of two-dimensional shapes. For example, a cube can be represented by the net in Figure 4.3.

Imagine folding the shape along each line. The resulting figure is a cube. Many people have difficulty visualizing this result without actually cutting it out and folding it, so I recommend you go through the process. You can also view an animation of it on YouTube at http://youtu.be/4mvJEn5Kst4.

A cube can also be represented by either of the nets shown in Figure 4.4. Try cutting these out and folding them to see how they become a cube.

Figure 4.3 A Net Representing a Cube

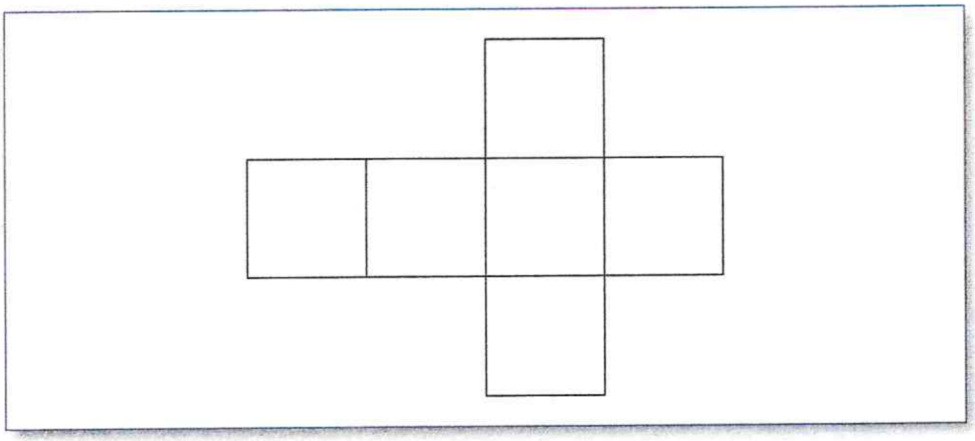
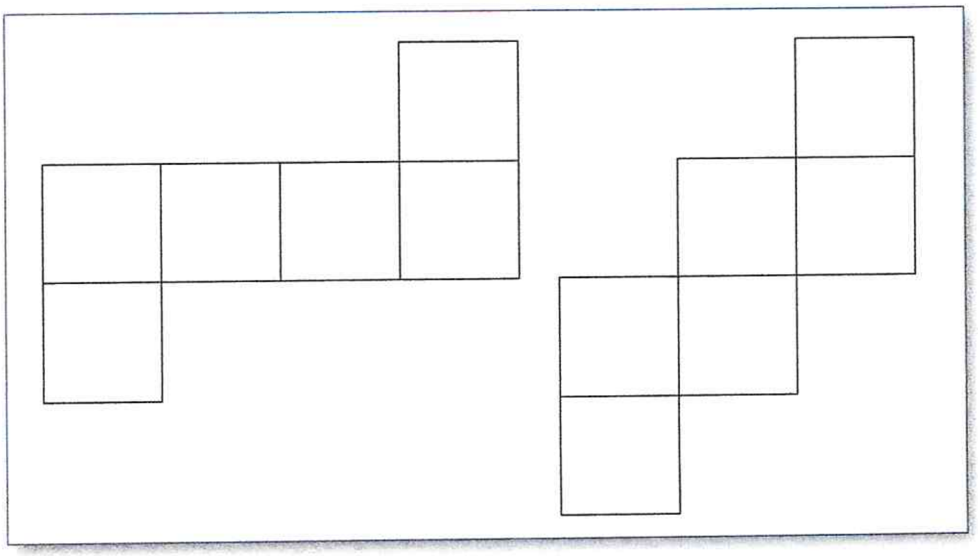


Figure 4.4 Two More Nets Which Form Cubes When Folded



Students have difficulty mentally manipulating these shapes unless they have extensive experience handling physical models and folding the nets. Asking the group to start with the box and determine how to generate a shape that can wrap it gives them experience with the actual manipulation which in turn helps them when they deal with the abstract nets later. Moreover, the students are also building a common language for expressing ideas about these shapes and about problem solving in general.

The conversation they're having is where the learning takes place. . . . It's the thinking they do while they're talking that leads to an understanding of the math.

I tried to reassure the anxious teacher. "The conversation they're having is where the learning takes place. Listen to what they're saying to each other. They are discussing options, debating what may or may not work, encouraging and helping and challenging each other. They're even arguing. It may seem disorganized, but it's the thinking they do while they're talking that leads to an understanding of the math."

"But, I feel so much pressure to cover all the content before the state test," she said, echoing what I hear from a lot of teachers when I talk to them about the 5 Principles.

I told her that I also felt the same pressure as a teacher. There's so much content to cram in before THE TEST that it's extremely tempting to take shortcuts and bypass the development of understanding. In this lesson, I spent an entire hour letting the students explore and discuss their solutions to this one problem. It took me another four hours of follow-up lessons to build enough experience and understanding with the boxes and paper and similar activities that they deeply understood nets and could fluently move from the concrete to the abstract. The students' conversations were crucial to that understanding and fluency.

"We know that task-related talking is important for learning the vocabulary of mathematics. Providing students the opportunity to communicate their actions can clarify mathematical terms and phrases" (Sousa, 2008, p. 90). During that first lesson, students struggled to communicate some ideas. As I circulated, I provided some terminology to help. We also closed the first lesson with a discussion about the paper shapes they created. "There is a name for this shape. Mathematicians call it a *net*." Students then wrote their own definitions based on their experience that day. Though these were far from complete, they formed the basis of our conversation over the next several days as we refined and clarified those definitions.

As you encourage these mathematical conversations in your classroom, keep these tips in mind.

1. **Give students abundant opportunities for casual math talk.** Conversation doesn't have to be structured to be productive. Johnson (2010) shares a case study of a molecular biology lab. Although most people would expect the major scientific breakthroughs to occur through lab experiments, "most important ideas emerged during regular lab meetings, where a dozen or so researchers would gather and informally present and discuss their latest work" (Johnson, 2010, p. 61).

2. **Guide conversations through targeted questions and modeling.** Avoid whole group instruction about how to talk about math. Instead, monitor student interactions, joining their discussions and showing them how you think and talk about math. Share insights, use appropriate math vocabulary, and ask specific questions designed to elicit thoughtful conversation.

3. **Provide students with conversation guides to help them when the discussion runs dry.** Figure 4.5 has a list of some good questions and discussion prompts to help you develop a conversation guide for your own classroom. These could be general guides for students to keep handy at all times, but for larger more complex problems, I recommend you develop a specific guide for the problem, just as if you were teaching a novel.

Figure 4.5 Sample Conversation Guide

Below are some questions and discussion prompts you might use to build a math conversation guide for your classroom:

1. What is easy about this problem? Why do you feel it is easy? Do you agree with your partners?
2. What do you think is confusing or complicated about this problem? Why is it confusing or complicated? Do you agree with your partners?
3. What happens if you try the opposite of what you're doing now? It may not work, but it could give you some interesting ideas about the problem.
4. If you gave this problem to an expert mathematician, what do you think he or she might try next? Why do you think that? What happens if you try the same thing?
5. Do you think you are missing some important information or knowledge? What else do you need to learn in order to make progress? Where (or from whom) do you think you could learn it?
6. Tell your partner your craziest idea for what to do next. Or, tell your partner why you think his or her crazy idea might work. Then try it. Talk about what you notice.
7. When you disagree with your partner, try switching sides. List as many reasons as possible why his or her idea could work and yours might not. Then listen while your partner tells you all the reasons why your idea could work. Talk about what you heard and learned.

Digital Tools and Resources for Communication in 2–5

While conversations about math in the classroom tend to be mostly verbal, students in Grades 2–5 can begin using online modes of discussion. Here are two tools that support online conversations about math.

Edmodo. Edmodo (http://www.edmodo.com) is a safe, closed environment where teachers can post questions and topics and students can respond online. Only those with an invitation code approved by the teacher can participate, so the teacher has complete control over participants in the discussion. The site also allows for a parent code so that you can invite parents to observe or even join the conversation.

Google Classroom. Google Classroom (http://classroom.google.com) is a newer entry into the online classroom space. It has fewer features than Edmodo, which can make it simpler for younger students to use and understand. Teachers can post announcements and assignments, and students can submit them online using their Google Drive. Teachers can also allow students to comment and create posts of their own, fostering discussion.

GRADES 4–7

Moving into the middle grades, students are ready to develop strong argumentation skills. A particularly valuable strategy is to ask students to convince you that their solution to a problem is a good one. But, don't be too easy to convince.

Convince Me

MP3, MP6, and MP7

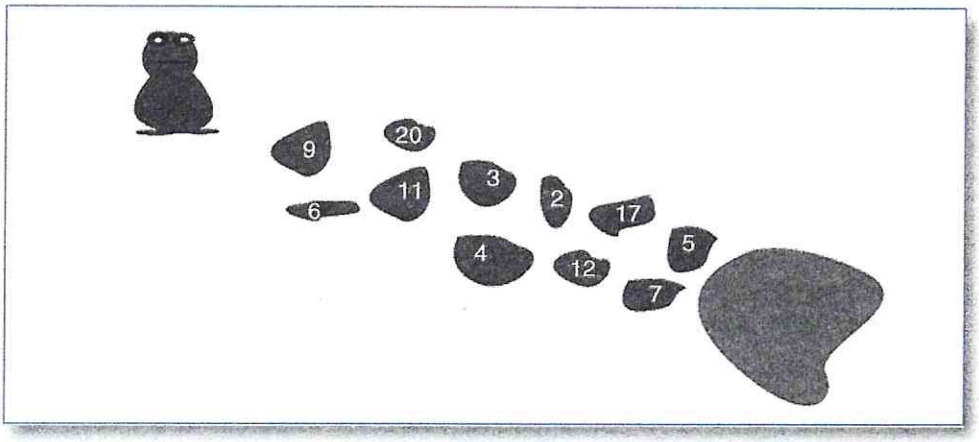
Mason, Burton, and Stacey (2010) present a 3–stage approach to building a convincing argument and exercising students' skills in communicating clearly and effectively about mathematical ideas. I recommend building up to the third stage over time. Allow students to practice Stage 1 for a while before you introduce Stage 2. Once they are comfortable with Stage 2, then you can introduce the more challenging Stage 3.

Stage 1: Convince Yourself. Begin by having students write down a few ideas about why they believe their solution to a problem is good. Encourage them to focus on their reasons for selecting each step, and how they know it works.

Example: Consider the following problem:

A frog jumped on several stones on his way to the pond. He did not land on the same stone twice. The product of all of the stones that he hopped on was 19,635. On which stones did he jump? (Holtz & Malen, n.d.)

Figure 4.6 Frog Pond Problem



Source: Holtz & Malen (n.d.). Courtesy The Franklin Institute.

Here is how one student responded with her reasons for convincing herself in bold:

- I divided 19,635 by 5, because **the last number is a 5 which means it could be divided by 5.**
- I divided 3,927 by 3, because **if you add all its digits 3 + 9 + 2 + 7 = 21 which is divided by 3.**
- I started dividing by the rest of the numbers until I got the right answer.
- I continued dividing until I got 2 of the numbers that were on the graphic.

Stage 2: Convince a Friend. In Stage 2, students begin to share their ideas with a friend, usually someone they worked with to solve the problem. In this case, their friend is someone who trusts them and their thinking, so the person is not difficult to convince. Most of the work here is on clear communication: expressing ideas in a way that someone else can understand them.

Example: Surina shares her frog pond solution with a classmate, Lamont. He has also worked on this problem, though not together with Surina. The conversation might go as follows:

Lamont: "You said the last number is a 5, but there is also a 7 on the other stone right next to the end."

Surina: "Oh, no, I didn't mean the last stone was a 5, I meant the last number in the answer was a 5. That means I can divide it by 5, so the frog must have landed on the 5 stone."

Lamont: "Oh, I get it now. But, I don't understand where you got the number 3,927. That's not in the problem."

Surina: "That's what you get after you divide 19,635 by 5. I guess I should have put that down."

Stage 2.5: Convince a Mathematical Friend. To take this a bit further and challenge students who master it more easily, try what I call "Stage 2.5." Encourage them to convince a "mathematical friend": someone who trusts them, but really understands math well and wants mathematical explanations for everything. This requires more precise language and more thoughtful use of math ideas.

Example: You listen to the exchange above and encourage Surina to be more precise in her explanation:

Surina: "OK, so what I meant was that the last digit in the final product from the problem was a 5. That means the product is divisible by 5 and the frog must have landed on that stone."

Stage 3: Convince an Enemy. The most sophisticated kind of argument—and the most difficult to construct—is one designed to convince someone skeptical, a person actively looking to pick apart your reasoning and find its flaws.

Initially, the teacher should take this role. Let students know what to expect: I am looking for ways to challenge your thinking. It isn't to make you think you're wrong, but to make sure you have thought about every possibility and have good evidence for what you're saying. Sometimes this makes you go back and revise your plan. That's okay! You will have a better solution and a more convincing argument in the end.

Example: You now step into Surina's and Lamont's conversation as the skeptic:

You: "OK, so I understand your first two steps, but then you just say you started dividing by the rest of the numbers. Did you just pick arbitrary numbers, or did you think about which one to try next?"

Surina: "Well, after I divided 3,927 by 3, I had 1,309. That isn't even, so the frog couldn't have jumped on any even numbers."

You: "How do you know that?"

Surina: "Because as soon as you multiply by an even number, the product will be even."

You: "OK, that makes sense. So why in the last step did you say you had to find 2 numbers that were on the graphic? Is there a specific reason it had to be two?"

And so on. For students to succeed in Stage 3, they need to not only be thorough and precise in their reasoning and language, but they need to anticipate objections and fully understand their math work in order to withstand challenges. Eventually, they will prepare more of their work up front, making sure their reasoning is airtight before they make it public.

Once the students master anticipating the challenges, students can then begin to take on this role for each other, playing devil's advocate against their partners. The ultimate expression of this kind of argumentation is to persuade a skeptic that *someone else's* reasoning is valid. To do this, students need to be able to comprehend and decipher another person's work and thought processes. In our scenario, Surina might give her revised solution to Nadav, who will then have to defend that work to Emma without coming back to Surina first for help.

Use Your Words

MP2, MP3, MP4, MP6, and MP8

Verbal conversations about math are crucial to developing good communication skills. Nevertheless, to take their work to the next level, students need to write about it. With a verbal conversation, there is opportunity for give-and-take, providing feedback to the speaker to clarify statements and fill in gaps in logic. When students write about a solution, they have to consider all relevant information up front, crafting a response that stands on its own.

For more than twenty years, Dr. Annalisa Crannell, mathematics department chair at Franklin and Marshall University, has been asking her college students to write their answers. In her course materials, she includes this explanation:

> Professional mathematicians spend most of their time writing: communicating with colleagues, applying for grants, publishing papers, writing memos and syllabi. . . . It is ironic, but true that most mathematicians spend more time writing than they spend doing math.

> But most of all, one of the simplest reasons for writing in a math class is that writing helps you to learn mathematics better. By explaining a difficult concept to other people, you end up explaining it to yourself. (Crannell, 1994, section 2)

"One of the simplest reasons for writing in a math class is that writing helps you to learn mathematics better." (Dr. Annalisa Crannell)

Students must have daily opportunities to write about math. Much of this writing should be focused on directly communicating about problems they are solving and mathematical arguments they are making, but don't limit students to these genres. Nearly any writing form and activity can be used to support and enrich mathematical learning. Here are a few suggestions:

- **Quickwrites.** Ask students at the beginning of a lesson or unit to take one minute to write down everything they think they know about the topic. The goal is to get as much down and fill up as much space as possible. This helps build student fluency with communicating math ideas and using appropriate vocabulary.

- **Individual Whiteboards.** Elementary teachers are very familiar with using individual response whiteboards for allowing all students to complete a math exercise. They promote student engagement and accountability, and allow teachers to make a quick formative assessment. Try using these for brief writing tasks as well, such as defining a key term, describing the next step in a solution, explaining a process, or writing an answer to a teacher question.

- **Math Reporter.** Have students work in pairs to write solutions and explanations to problems. One student takes the role of mathematician, and the other student is the math reporter. The mathematician verbally explains the process and steps in a solution, and the reporter takes notes, ask questions, and collects information. The reporter then writes a news article on the problem.

- **Exit Tickets.** On the way out the door, have students hand you a one-sentence summary of the math they used, an idea they learned, or a vocabulary definition.

- **Math Fiction.** Don't ignore the possibility of combining math and fiction. There are many examples of children's and young adult literature that use mathematical ideas as a launchpad for intriguing stories. Middle grade students enjoy writing books for young children. Why not ask them to teach an interesting math idea through a story?

- **Math Poetry.** Poetry lends itself naturally to math connections. The rhythms and patterns are obvious ways to work math into student writing, but students can also express math ideas in poetry. For example, challenge students to write poems about circles that use the digits in pi as a template for the number of words or syllables in each line.

- **Math Research.** So often we focus entirely on math content in our math classes while ignoring the rich history and fascinating personalities of mathematics. Have students research the origins of our number system, or investigate ways that math was used in events they are studying in social studies. Encourage them to find out about the people who originally invented or proved the math concepts they are learning. There are abundant sources of information about math history and famous mathematicians on the Internet. Pay special attention to women and people of color to give your students diverse, positive mathematical role models.