Chapter 2
Working with the Big Ideas in Number and the Australian Curriculum: Mathematics

Dianne Siemon
John Bleckly
Denise Neal

This chapter will explore why big ideas have become a topic of interest in mathematics education, what these might look like in relation to the teaching and learning of number in the early to the middle years of schooling, and how these are reflected in the *Australian Curriculum: Mathematics*. In particular, it will consider those ideas and strategies without which student's progress in mathematics will be seriously impacted. Identified as trusting the count, place-value, multiplicative thinking, partitioning, and proportional reasoning (Siemon, 2006), each idea will be explored through the lens of a set of diagnostic materials that have been found to be effective in mainstream and remote Indigenous settings. Two case studies will be reported to demonstrate the efficacy of the diagnostic tools and their associated advice in helping teachers identify and respond to the specific learning needs of their students while building a deeper understanding of the mathematics needed for teaching.

The crowded curriculum and the lack of succinct, unambiguous guidelines about the key ideas and strategies needed to make progress in school mathematics have long been a concern of teachers. This is particularly the case for Number which is the area most responsible for the significant range in mathematics achievement in the middle years of schooling (Siemon, Virgona & Corneille, 2001; Siemon, Breed, Dole, Izard & Virgona, 2006). While the importance of focussing on the 'big ideas' is widely recognised (e.g., Charles, 2005; Kuntze, Lerman, Murphy, Kurz-Milcke, Siller & Winbourne, 2009; National Curriculum Board [NCB], 2009: Ontario Ministry of Education, 2006), there is little agreement about what these ideas are and how they are best represented to support the teaching and learning of mathematics in schools.

The development of the *Australian Curriculum: Mathematics* [ACM] (Australian Curriculum Assessment Reporting Authority, 2011) provided an important opportunity to negotiate and articulate the big ideas in school mathematics (see

In B. Atweh, M. Goos, R. Jorgensen & D. Siemon, (Eds.). (2012). Engaging the Australian National Curriculum: Mathematics – Perspectives from the Field. Online Publication: Mathematics Education Research Group of Australasia pp. 19-45.

Siemon, 2011b). The extent to which this has been achieved is discussed in terms of five big ideas in Number that have been found to be useful in identifying learning needs and informing teacher's responses to those needs in Victorian, Tasmanian and South Australian schools.

Why a Focus on Big Ideas?

> Students need to learn mathematics in ways that enable them to recognise when mathematics might help to interpret information or solve practical problems, apply their knowledge appropriately in contexts where they will have to use mathematical reasoning processes, choose mathematics that makes sense in the circumstances, make assumptions, resolve ambiguity and judge what is reasonable in the context. (Commonwealth of Australia, 2008, p. 11)

Evidence from the 2009 *Programme for International Student Assessment* [PISA] shows that there has been a significant decline in the proportion of Australian students reaching Level 5 or above on the mathematical literacy scale (Thomson, Bortoli, Nicholas, Hillman & Buckley, 2011). This is consistent with data from the *Middle Years Numeracy Research Project* [MYNRP] which found that a significant proportion of students in Years 5 to 9 experience considerable difficulty interpreting problem situations, applying what they know to solve unfamiliar situations, explaining their thinking and communicating mathematically (e.g., Siemon, Virgona & Corneille, 2001).

While it is difficult to argue cause and effect at this macro level, there is little doubt that the opportunity to engage in sustained problem solving and in-depth investigations is significantly influenced by the amount of content that teachers feel they have to teach and how that content is offered (NCB, 2009). This is reflected in the size and organisation of mathematics textbooks in the middle years where mathematics is typically presented as a set of disconnected topics and the primary mode of learning is example-practice-practice. The fact that there is considerable overlap in the content of such texts and the vast majority of problems tend to be relatively low-level, skill-based repetitious exercises (e.g., Vincent & Stacey, 2008) is unlikely to be conducive to learning mathematics in the way suggested by the *National Numeracy Review* (Commonwealth of Australia, 2008). A focus on the big ideas is needed to 'thin out' the over-crowded curriculum (NCB, 2009; National Mathematics Advisory Panel, 2008) and create opportunities to rethink and transform existing approaches to the teaching and learning of mathematics.

> Teachers need to understand the big ideas of mathematics and be able to represent mathematics as a coherent and connected enterprise. (National Council of Teachers of Mathematics, 2000, p. 17)

A focus on big ideas and the links between them is also needed to strengthen student understanding and help deepen teacher knowledge and confidence for teaching mathematics (Charles, 2005), the importance of which has been demonstrated by research on the characteristics of effective teachers of mathematics (e.g., Askew, 1999; Charles, 2005; Clarke & Clarke, 2002; Hattie, 2003; Ma, 1999). For instance, effective teachers recognise the connections between different aspects and representations of mathematics. They ask timely and appropriate questions, facilitate and maintain high-level conversations about important mathematics, evaluate and respond to student thinking during instruction, promote understanding, help students make connections, and target teaching to ensure key ideas and strategies are understood. "A clearly, succinctly written curriculum will assist this" (NCB, 2009, p. 12).

What is a 'Big Idea' in School Mathematics?

The content of school mathematics has always been subjected to some form of categorisation. In recent times, these categorisations have included process as well as content strands, for example, the *National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991). For its purposes, PISA categorised school mathematics in terms of 'quantity, space and shape, and uncertainty'. In the *Discussion Paper on School Mathematics for the 21st Century*, the Australian Association of Mathematics Teachers (2009) added 'variables, relationships and change' to the PISA list but also included four 'mathematical actions'. The ACM is organised in terms of three content strands – 'Number and Algebra, Measurement and Geometry, and Statistics and Probability' - and four proficiencies. While some of these categories (e.g., quantity, uncertainty) might be regarded as really big ideas in school mathematics, they are too broad to inform teacher's everyday practice. A more refined set of key ideas and strategies and the links between them is needed to inform teaching and scaffold student learning.

Charles (2005) defines a 'big idea' as "a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole" (p. 10). For example, "Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value" (p. 14) is a statement of a big idea. However, while he identifies twenty-one 'Big Ideas' in mathematics and provides 'examples of mathematical understandings' for each, no claims are made about possible learning progressions or developmental priorities beyond what is loosely and perhaps unintentionally implied by the organization of the list. For example,

> Big Idea #2: The base ten numeration system is a scheme for recording numbers using digits 0-9, groups of ten, and place value. … (p. 13)

> Big Idea #13: Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found. (p. 18)

While not defining a 'big idea', the Ontario Ministry of Education (2005) lists five 'big ideas' in number sense and numeration for K to 3 – 'counting, operational sense, quantity, relationships, and representations'. For Years 4 to 6 'counting' is replaced by 'proportional reasoning' and in Years 7 to 8, the list is reduced to 'quantity relationships, operational sense, and proportional relationships'. This approach provides some indication of the developmental progressions involved but these are not specifically delineated.

More recently, the *Awareness of the Big Ideas in Mathematics Classrooms* project (Kuntze, Lerman, Murphy, Siller et al., 2009), which is aimed at "encouraging teachers' reflections on overarching concepts in mathematics and on their potential for learning" (p. 9), has identified four characteristics of big ideas. These can be summarised as ideas that have high potential for building conceptual understanding, meta-knowledge about mathematics as a science, meaningful communication strategies, and professional reflection. Examples of big ideas from this standpoint include 'using multiple representations', 'giving arguments or proving' and 'dealing with infinity'. While these are undoubtedly important indicators of mathematical reasoning and best teaching practice, it is not clear how these translate to learning trajectories that could be used to inform teaching and support mathematics learning over time.

For the purposes of the *Assessment for Common Misunderstandings* (Siemon, 2006) and the *Developmental Maps* (Siemon, 2011a) which were developed for the Victorian Department of Education and Early Childhood Development [DEECD], a 'big idea' in mathematics:

- is an idea, strategy, or way of thinking about some key aspect of mathematics without which, students' progress in mathematics will be seriously impacted;
- encompasses and connects many other ideas and strategies;
- serves as an *idealised cognitive model* (Lakoff, 1987), that is, it provides an organising structure or a frame of reference that supports further learning and generalizations;
- cannot be clearly defined but can be observed in activity … (Siemon, 2006, 2011a).

Why Big Ideas in Number?

Teachers routinely point to Number as the most difficult aspect of the school mathematics to teach and learn. This is reflected in the time spent on number in the school mathematics curriculum and evident in the data from the MYNRP, which used rich assessment tasks and partial credit items to explore number sense, measurement and data sense, and space sense in a structured sample of 6859 Year 5 to 9 students in 1999-2001 (Siemon, Virgona & Corneille, 2001). The results of this large-scale study found that there was as much difference in numeracy achievement within schools as between schools, that in any one year level there was there was up to an 8 year range in ability, and the needs of 'at risk' learners were not being met. A key finding of the MYNRP was that the differences in performance were almost entirely due to difficulties with larger whole numbers, decimals, fractions, multiplication and division, and proportional reasoning, collectively recognised as multiplicative thinking (Vergnaud, 1983).

As a consequence, the *Scaffolding Numeracy in the Middle Years* [SNMY] project was designed to explore the development of multiplicative thinking in Years 4 to 8 using rich tasks and partial credit items. Rasch modeling (e.g., Bond & Fox, 2001) was used to analyse the responses of just under 3200 students in three school clusters (one secondary school and three or more associated primary schools), two in Victoria and one in Tasmania (Siemon, Breed et al., 2006). A *Learning and Assessment Framework for Multiplicative Thinking* [LAF] was identified on the basis of this analysis comprising eight hierarchical zones ranging from additive, count-all strategies (Zone 1) to the sophisticated use of proportional reasoning (Zone 8) with multiplicative thinking not evident on a consistent basis until Zone 4. The proportion of students by Zone by Year level is shown in Figure 1.

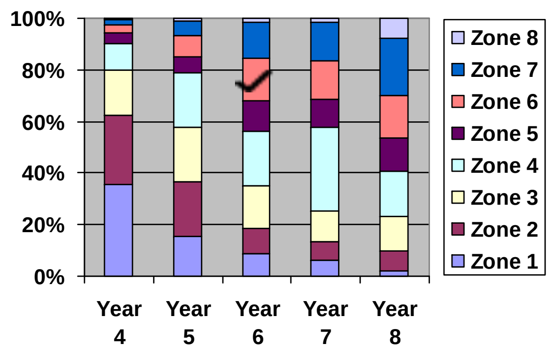

Figure 1. Proportions of students by Zone and Year Level from the initial phase of SNMY Project

The results of the SNMY confirmed the finding of the MYNRP that there was an 8 year range in achievement at each year level and when the LAF Zones were analysed against curriculum expectations, it was evident that up to 40% of Year 7

and 8 students performed below curriculum expectations and at least 25% were well below expected level (Siemon, Breed et al., 2006).

This discrepancy is unacceptable in a country that prides itself on providing opportunities for all (Ministerial Council on Education, Employment, Training & Youth Affairs, 2008). Multiplicative thinking is a key indicator of success in school mathematics in the middle years and as such it is imperative that the key ideas and strategies that underpin the transition from additive to multiplicative thinking are clearly articulated and understood by teachers and curriculum developers. A focus on the big ideas in number is essential to inform more targeted approaches to the teaching and learning of mathematics to ensure that all students have the opportunity to deepen their understanding and participate fully and effectively in school mathematics.

Big Ideas in Number and the Australian Curriculum: Mathematics

Scaffolding student learning is the primary task of teachers of mathematics. However, this cannot be achieved without accurate information about what each student knows already and what might be within the student's grasp with some support from the teacher and/or peers. This not only requires a clear understanding of the key ideas, representations and strategies in school mathematics, how they are connected and how they might be acquired over time, it also requires assessment techniques that expose student thinking, interpretations of what different student responses might mean, and some practical ideas to address the particular learning needs identified (Siemon, 2006). As we have seen above, this is particularly important in relation to a relatively small number of 'big ideas' and strategies in Number.

The *Assessment for Common Misunderstanding* tools [hereinafter referred to as the tools] were developed for the Victorian Department of Education and Early Childhood Development (see Siemon, 2006) to address this need. They draw on research-based tasks and represent what Callingham (2011) has referred to as productive assessment, in that they provide useful, timely, appropriate information fit for purpose. Based on earlier work with pre-service teachers and schools in the Northern Territory (Siemon, Enilane & McCarthy, 2004), the tools were developed to help teachers better "understand and monitor their individual students' developing strategies and particular learning needs" (National Curriculum Board, 2008, p. xiv) in relation to a small number of very big ideas in Number without which student's progress in mathematics will be severely restricted. These ideas are summarised in Table 1. The first five ideas are then considered in terms of their associated tools and the ACM (version 1.2) as it is these ideas that most concern the development of multiplicative thinking in the middle years of schooling.

Table 1. Big ideas in Number by stages of schooling (Siemon, 2006)

By the end of:	Big Idea	Indicated by:
Foundation Year	Trusting the Count	Access to flexible mental objects for the numbers to ten based on part-part-whole knowledge derived from subitising and counting (e.g., know that 7 is 1 more than 6, 1 less than 8, 5 and 2, 2 and 5, 3 and 4 without having to make or count a collection of 7)
Year 2	Place-value	Capacity to recognise and work with place-value units and view larger numbers as counts of these units rather than collections of ones (e.g., able to count forwards and backwards in place-value units)
Year 4	Multiplicative Thinking	Capacity to work flexibly with both the number in each group and the number of groups (e.g., can view 6 eights as 5 eights and 1 more eight). Recognises and works with multiple representations of multiplication and division (e.g., arrays, regions and 'times as many' or 'for each' idea).
Year 6	(Multiplicative) Partitioning	Ability to partition quantities and representations equally using multiplicative reasoning (e.g., a fifth is smaller than a quarter, estimate 1 fifth on this basis then halve and halve remaining part again to represent fifths), recognise that partitioning distributes over previous acts of partitioning and that numbers can be divided to create new numbers
Year 8	Proportional Reasoning	Ability to recognise and work with an extended range of concepts for multiplication and division including rate, ratio, percent, and the 'for each' idea, and work with relationships between relationships
Year 10	Generalising	Capacity to recognise and represent patterns and relationships in multiple ways including symbolic expressions, devise and apply general rules

The following descriptions draw on material written for the DEECD in 2006 and 2011 (Siemon, 2006, 2011a) and Siemon, Beswick, Brady, Clark, Faragher & Warren (2011). They comprise a number of easy to administer, performance-based tasks designed to address a key area of Number at different levels of schooling from Foundations to the end of Year 10. In the associated teaching advice a range of student responses is identified for each task and, for each of these, an interpretation of what the response implies is provided together with targeted teaching suggestions.

Trusting the Count

The term 'trusting the count' was originally proposed by Willis (2002) to draw attention to the fact that children may not believe that if they counted the same collection again they would arrive at the same amount. More recently this term has been appropriated and extended to refer not only to the belief that counting will produce an invariant result (literal interpretation), but also to the capacity to access mental objects for the numbers to ten that render counting unnecessary in most dealings with those numbers (Siemon et al., 2011). Derived primarily from extensive experiences with subitising (the ability to recognise small collections without counting), a child trusts the count for a number such as 8 when he or she can access a repertoire of knowledge items and images for 'eightness' that obviates the need to represent and count collections of 8 in order to work with 8. Viewed in this way, trusting the count also supports a sense of numbers beyond ten, for example, a collection of 16 can be recognised as 1 ten and 6 more without counting on by ones (Siemon, 2006).

Trusting the count is a big idea that builds on and connects early number ideas derived from counting and subitising. In particular, it presumes children are familiar with the number naming sequence and understand what is meant by *more, less* and *the same* in this context. Trusting the count is not about addition or subtraction, although it is a key component of additive thinking. It is about deeply understanding what each of the numbers to ten means and the various ways in which they might be represented in terms of their parts. It is an essential pre-requisite for understanding larger numbers and developing a sense of quantitative reasoning (Smith & Thompson, 2007).

Two tools are used to evaluate children's capacity to trust the count (see Siemon, 2006). The first assesses children's capacity to recognise numbers to 5 without counting and on this basis to recognise the remaining numbers to ten without counting referred to as *conceptual subitising* by Clements and Samara (2007). The second tool is based on a task developed by Steffe and his colleagues in the early 1980s to evaluate children's counting strategies (Steffe, Cobb & von Glasersfeld, 1988). It is used in this context to examine the extent to which children have access to mental objects for the numbers to ten.

The *Australian Curriculum: Mathematics* [ACM] at this level of schooling refers to the 'language and processes of counting' (ACMNA001) and the ability to 'connect number names, numerals and quantities (ACMNA002), 'subitise small collections' (ACMNA003) and 'compare, order and make correspondences between collections' (ACMNA289). While these capacities are necessary to build mental objects for each of the numbers to ten, they are not sufficient. By the end of their first 12 to 18 months of school, children need a deep understanding of the numbers to 10 that goes beyond the language and processes of counting to ensure that when they hear, read, write, or say a number such as 'seven', they can imagine that number in terms of its parts (e.g., as 1 more than 6, 5 and 2, or 3 and 4) and how it relates to other numbers (e.g., as 1 less than 8 or 3 less than 10), without having to make, count or literally 'see' a collection of 7 objects. A deliberate and explicit focus on the development of *part-part-whole knowledge* is needed to ensure that children move beyond the language and processes of counting to develop mental objects for each of the numbers to ten that they can use flexibly without recourse to materials or models.

Place Value

The big idea of place-value in the early years of schooling is that it provides a system of new units based on the notion that '10 of these is 1 of those' that can be used to work with and think about larger whole numbers in efficient and flexible ways. By the end of their third year of school (generally Year 2), most students can count by ones to 100 and beyond, read and write numbers to 1000, orally skip count by twos, fives and tens, and identify place-value parts (e.g., they can say that there are 4 hundreds 6 tens and 8 ones in 468). However, these behaviours do not necessarily mean that children understand place-value as many students still think about or imagine these numbers as collections of ones, and they are unable to rename numbers in terms of their place-value parts (e.g., rename 476 as 47 tens and 6 ones) or work in place-value parts (e.g., name the number 2 tens less than 5308). That is, they do not recognise tens, and hundreds as units within a larger, place-based system of numeration.

Four tools are used to evaluate the extent to which children understand place-value. The first, the Number Naming Tool is based on a task used by Ross (1989) and explores the meanings children attach to 2-digit numerals (e.g., the meaning of 6 and 2 in 26) and the extent to which they can be distracted by regrouping 26 counters into groups of 4 (i.e., they understand ten as a countable unit). The Efficient Counting Tool indicates the extent to which students can use twos, fives or tens as countable units to count large collections more efficiently. The Sequencing Tool explores the strategies students use to locate a 2-digit number on a 0-100 number line and the Renaming and Counting Tool examines student's capacity to name and rename a 3-digit number and count forwards and backwards in place-value parts from a given 4-digit number.

In the ACM, although number and place-value is used as a thread across all year levels there are only four references to place-value in the content descriptions, one at Year 1, one at Year 3 and two at Year 4. The first, "count collections to 100 by partitioning numbers using place value" (ACMNA014), can be accomplished

without recognising tens as units (e.g., 'partitioning' 43 as 40 and 3 does not emphasise ten as a countable unit as 40 is the name for 40 ones). The next two descriptors are concerned with assisting calculations (ACMNA053 and ACMNA073) and the fourth is concerned with the extension of the place-value system "to tenths and hundredths" (ACMNA079). Number sequences and skip counting are variously referred to in Years 1 and 2 (e.g., ACMNA012 and ACMNA026) but counting by twos, threes, fives or tens does not necessarily mean that 2, 3, 5 and 10 are understood as countable units. A count of 3, 6, 9, 12, … or 10, 20, 30, 40, … could simply be seen as a shortened form of counting by ones.

The ability to "recognise, model, read, write, and order numbers to at least 100" (ACMNA013), "to at least 1000" (ACMNA027), "to at least 10 000 (ACMNA052), and "to at least tens of thousands" (ACMNA072) are necessary pre-requisites for working with larger numbers but again, these capacities do not necessarily mean that children understand the structural basis of the base ten system of numeration or recognise tens, hundreds, and thousands as abstract composite units (Siemon et al., 2011).

Additive thinking is not regarded as a big idea in its own right as it builds upon the two ideas of trusting the count and place value (Siemon et al., 2011). It is evident when children work with numbers as mental objects and rename numbers as necessary to facilitate calculations. For example, asked to calculate 36 and 27, an accomplished additive thinker might draw on her knowledge of place value to recognise this sum as 5 tens and 13 ones and therefore 63. Alternatively, she might add 2 tens to 36 to get 56 then, recognising 7 as 4 and 3, add 4 to 60 then 3 more to arrive at 63.

Multiplicative Thinking

For the purposes of the SNMY project, multiplicative thinking was described in terms of:

- a capacity to work flexibly and efficiently with an extended range of numbers (and the relationships between them);
- an ability to recognise and solve a range of problems involving multiplication and/or division including direct and indirect proportion; and
- the means to communicate this effectively in a variety of ways (e.g., words, diagrams, symbolic expressions, and written algorithms) (Siemon, Breed et al., 2006)

Multiplicative thinking is a critically important 'big idea' as it underpins virtually all of the work in number and algebra in the middle years of schooling. By the end of Year 4 students need to be able to think about multiplication in a number of different ways so they can recognise when multiplication is required and how it relates to division, develop efficient mental strategies and meaningful forms of written computation to solve a wider range of problems, and make connections to fraction representations, percent, rate and ratio. To achieve this they need to experience multiplication and division in ways that support a critical shift in thinking from a reliance on equal groups and repeated addition to a more general understanding of multiplication and division in terms of factor-factor-product (Siemon et al., 2011).

Six tools are provided to evaluate multiplicative thinking at this level. These are summarised in Table 2.

Table 2. Tools used to examine the emergence of multiplicative thinking (Siemon, 2006)

Tool	Designed to evaluate student's capacity to:
Additive strategies	access to mental objects for the numbers to ten and efficient mental strategies for addition and subtraction
Countable units	recognise numbers as abstract composite wholes (Killion, Steffe & Stanic, 1989), that is, as countable units in the absence of physical materials/models
Sharing	share equally, recognise commutativity (e.g., that 3 groups of 4 is the same as 4 groups of 3), appreciate the meaning of 'times as many as'
Array and region	use the properties of arrays and regions to determine the total amount without counting by ones or skip counting
Cartesian product	solve problems involving the Cartesian product or 'for each' idea of multiplication (e.g., the total number of lunch orders given three types of bread, 4 different fillings and 2 types of fruit)
Simple proportional reasoning	Use 'if … then' reasoning to solve simple proportional reasoning problems (Clarke & Kamii, 1996)

In the ACM, the only reference to any of these key ideas is in Foundations where sharing is mentioned (ACMNA004) and in Year 2 where students are expected to recognise and represent "multiplication as repeated addition, groups and arrays" (ACMNA031) and "division as grouping into equal sets" (ACMNA032). However, sharing a collection equally does not necessarily indicate multiplicative thinking unless students recognise the relationship between the dividend and the quotient (Nunes & Bryant, 1996) and working with arrays is no guarantee of multiplicative thinking either unless the focus of attention is shifted from a count of groups of the same size (additive) to a given number of groups of any size (Siemon et al., 2011). Importantly, the region idea is not mentioned at all and yet this underpins the 'area' or 'by' idea of multiplication (i.e., each part multiplied by every other part) which is needed to support the multiplication of larger whole numbers (e.g., 2-digit by 2-

digit multiplication), the interpretation of fraction diagrams (e.g., thirds by fifths are fifteenths) and ultimately the multiplication of linear factors.

In Year 3 students are expected to "recall multiplication facts of two, three, five and ten and related division facts" (ACMNA056). This wording together with the reference to number sequences "increasing and decreasing by twos, threes, fives and tens" (AMNA026) in Year 2 implies that the multiplication facts are learnt in sequence (e.g., 1 three, 2 threes, 3 threes, 4 threes, 5 threes etc) rather than on the basis of number of groups irrespective of size (e.g., 3 of anything is double the group and one more group). The references to "investigate number sequences involving multiples of 3, 4, 6, 7, 8, and 9" (ACMNA074) and "recalling multiplication tables" (Fluency proficiency) at Year 4 reinforce this observation. This is unfortunate given the reported success of alternate approaches to learning the multiplication facts based on commutativity and distributivity (not mentioned in the ACM until Year 7) and renaming numbers (e.g., McIntosh & Dole, 2004; Siemon et al., 2011).

Factors and multiples are referred to in Year 5 (ACMNA098), "properties of primes, composite, square and triangular numbers" in Year 6 (ACMNA122), indices in Years 7 and 8 (ACMNA149 & ACMNA182), and solving problems involving specified numbers and operations across year levels (e.g., ACMNA100, ACMNA101 and ACMNA103). These content descriptors vary little from the topic-based curriculum of 50 years ago. There is no suggestion of the connections between them or that something other than a repeated addition model of multiplication is needed to support a deep understanding of factors and indices (Confrey, Maloney, Nguyen, Mojica & Myers, 2009).

Partitioning

The idea that a collection or a quantity can be expressed in terms of its parts is fundamental to developing a strong sense of number. This can be done additively (as in part-part-whole knowledge and renaming whole numbers in terms of their place-value parts) or multiplicatively (as in the production of equal parts). To clarify this distinction, Confrey and her colleagues (Confrey, et al., 2009) introduced the term equipartitioning (or splitting) to refer to

> behaviors that create *equal-sized groups*. In addition, we would assert that division is most directly derived from equipartitioning, with multiplication following as its inverse, rather than the traditional view that multiplication precedes division. … Equipartitioning/splitting as an operation leads to partitive division as well as to multiplication. (p. 347)

Multiplicative partitioning, equipartitioning, or partitioning as it is used in this context is a 'big idea' that underpins the capacity to work meaningfully with rational numbers and their representations. In particular, to compare, order and rename fractions, build strong connections between multiplication, division, fractions and decimals, and support the extension of multiplicative thinking to rate, ratio and percent (Siemon et al., 2011).

Seven tools examine the following indicators of partitioning as it is used in this context.

- Distinguish between fraction and non-fraction representations in multiple settings.
- Recognise the relationship between the number of equal parts and the size and name of the parts (e.g., as the number of parts/shares increase the size of each part or share decreases).
- Use efficient multiplicative strategies to construct indicative fraction diagrams and line models, name and record common fractions and decimals
- Recognise that the relative magnitude of a fraction depends upon the relationship between the numerator ('how many') and the denominator ('how much').
- Use meaningful strategies to compare, order and rename common fractions and decimal fractions.

In the early years, the ACM refers to the capacity to "recognise and describe half as one of two equal pieces" (ACMNA016) and "to recognise and interpret common uses of halves, quarters and eighths of shapes and collections" (ACMNA033) but no mention is made of the important link to sharing which provides a powerful basis for the creation of equal parts and the link between fractions and partitive division (Nunes & Bryant, 1996). In Year 3, students are expected to be able to "model and represent unit fractions including 1/2, 1/4, 1/3, 1/5 and their multiples to a complete whole" (ACMNA058). This suggests that fraction symbols are expected at this stage, which is problematic given the well known difficulties associated with interpreting fraction symbols and representations (e.g., Lamon, 1999). Also, the reference to counting fractions at Year 4 (ACMNA078) appears to privilege the fraction as number or measure idea over the many other representations of fractions, for example, part-whole relations, quotients, ratios and operators (Confrey et al., 2009; Lamon 1999). Focussing on fractions as measures could also lead to an over-reliance on additive, whole number-based approaches to locating fractions on a number line at the expense of multiplicative approaches such as partitioning.

The inclusion of hundredths at Year 4 (ACMNA079) is mystifying in view of the research on decimal fraction misconceptions (e.g., Steinle & Stacey, 2004). It has possibly been included here because calculations to the nearest cent have been included at this level (ACMNA080) but there is little/no evidence to suggest that being able to work with money contributes to a deep understanding of decimal fractions. The fact that there is no mention of percentage benchmarks such as 50%,

25% at this level is also somewhat surprising given children's capacity work with repeated acts of halving (percentages are not referred to at all until Year 7).

Comparing and ordering "common unit fractions" and locating them on a number line (ACMNA102) is consistent with partitioning as is recognising "that the number system can be extended beyond hundredths" (ACMNA104). However, given that partitioning supports generalisations about how fractions might be renamed (e.g., if the total number of parts are increased by a certain factor then the number of parts required is also increased by that factor), it seems strange that the comparison of unlike fractions (fractions with unrelated denominators) has been pushed back to Year 7.

Proportional Reasoning

Proportional reasoning involves recognising and working with relationships between relationships (i.e., ratios) in different contexts. Proportional reasoning is important as it underpins the work done in other domains of mathematics (e.g. scale diagrams, the analysis of similar figures in geometry, and calculations involving percentages in financial mathematics) and provides a powerful basis for understanding functional relationships more generally.

The following indicators of proportional reasoning are examined by eight tools.

- Use relational thinking (multiplicative) as opposed to absolute thinking (additive) to analyse change over time or compare relationships.
- Identify and describe relationships between quantities in a range of problem contexts
- Work flexibly and confidently with the quantities involved (i.e., measures, rates and/or ratios expressed in terms of natural numbers, rational numbers, percents and/or integers).
- Use a scale factor to enlarge/reduce a 2-dimensional shape or estimate distances on a scale map.

The ACM does not refer to proportional reasoning explicitly until Year 9 where reference is made to solving problems involving direct proportion and simple rates (ACMNA208) and enlargements, similarity, ratios and scale factors in relation to geometrical reasoning (ACMMG220 & ACMMG221). While many of the prerequisite skills are included in Years 6 to 8, these appear in the form of disconnected and only slightly differentiated skills. For example, "find a simple fraction of a quantity" (ACMNA127) at Year 6, "express one quantity as a fraction of another", "find percentages of quantities and express one quantity as a percentage of another" (ACMNA 155, & ACMNA158) at Year 7, and solve a range of problems involving percentages, rates and ratios (ACMNA187 & ACMNA188) at Year 8.

Importantly, there is nothing to suggest how these skills relate to one another or their rich connections to multiplicative thinking more generally.

The Verdict

While it is still early days and it remains to be seen how educational systems and schools will work to bring the ACM to fruition, the casual reader could be forgiven for thinking that the ACM is just a thinner version of existing State and Territory mathematics curricula. The big ideas described here are not entirely absent from the ACM but they are not visible in ways that might provide "a powerful transformational force for deepening teacher knowledge for teaching mathematics and energising practice over time" (Siemon, 2011b, p. 68).

The proficiencies which value conceptual understanding alongside procedural fluency as well as mathematical reasoning and problem solving, offer some potential to 'connect the dots' but the description of these at each year level is very brief and it will require a significant commitment to teacher professional learning to achieve this. Recent experience points to the benefits of focussing on the big ideas and using these as an organising frame to target teaching to learning needs and improve student outcomes. Some of this experience is reported in what follows.

Working with the Big Ideas in Number

The following cases illustrate how the *Assessment for Common Misunderstandings* materials have been used in South Australia and Tasmania to promote teacher professional learning and inform teaching practice. The teacher's names have been used with permission and their quotes are included in italics to distinguish the teacher voice from other quotes included in the chapter.

The South Australian Experience

In August 2009 the South Australian Department of Education and Children's Services [DECS] introduced the Literacy and Numeracy National Partnership [LNNP] project which placed 14 Numeracy Coaches in primary schools. This initiative was funded through the Australian Government's Smarter Schools National Partnership. The Numeracy Coaches are each based in one or two schools and work with teachers and school leaders to improve numeracy outcomes across the school. Coaches are supported by an intensive professional learning program focussing predominantly on the skills of coaching, mathematical pedagogical content knowledge, working with student achievement data, and whole school improvement.

The *Assessment for Common Misunderstanding* materials were used with permission from DEECD as the basis for the coaching initiative and coaches were provided with a hard copy of these materials which were referred to as the *Big Ideas in Number*. Extensive professional learning on the big ideas was provided to the numeracy coaches who then worked in their schools with teachers to implement these ideas and strategies in their classrooms. As part of the LNNP evaluation, coaches were interviewed about the impact and outcomes of the project. The following examples are illustrative of their feedback.

Brianna is the LNNP Numeracy Coach at a rural primary school and she has been working closely with Emily and her Reception/Year 1 (R/1) class on the big idea of Trusting the Count. Previously Emily had a strong focus on children being able to count and match number symbols to collections and number names. She now recognises that this was important but not enough. She believes that the increased emphasis on developing children's deep understandings of the numbers 0 to 9 and particularly their part-part-whole understanding (i.e., that seven is five and two, three and four and so on), has paid dividends as children can work flexibly with numbers from the earliest years. Her children don't just learn what seven is but also how to break seven into its parts and put them back together again!

In this class Emily and Brianna have used a wide range of materials, such as subitising cards, ten frames, dice, and clothes lines to develop understanding. They have also explored electronic technologies like Bee Bots, Interactive whiteboards and iPods to support learning. The children program Bee Bots to move a given number of steps along a number line, guessing where it will end up or matching the numeral to the number line position. They talk about 'how many more steps to ten'. They develop their understanding of doubles by playing a doubles dice game on the interactive white board. In this game a die is 'rolled' and the outcome displayed on the board. The children have to double the number rolled and then select the answer from a line of numbers from 1 to 12. They get one point for each correct answer and have a time limit of 60 seconds to get the highest score they can. Emily then records this score for each child to map their improvement over time. Some of the children can double and find the answer in the line up so quickly that visiting adults can't match the student's score. The iPod Touches have been a really big hit with the children. They have quickly mastered the navigation and use of the visual menus. Emily and Brianna have found some age appropriate iPod apps that focus on early number skills and they continue to look for more.

Over the time that Emily has been working with the big ideas in number she has learnt a lot about how children learn number concepts. She expresses her own learning as,

> I am able to make the learning more hands on and by watching children engaged in learning activities I can often 'see' what they are thinking. I have a better grasp of what I'm looking for and my own understanding of conceptual development in the number strand continues to grow. I can now target children's learning more accurately to their needs so there is less maths time wasted.

Madeline, also working with numeracy coach Brianna, found the big ideas really helped her to change her approach to teaching multiplication facts to her Year 3/4 class. She now emphasises a strategies approach rather than relying on rote learning – the students know, for example, that 5 times a number is 'half of ten of it' or that to find three times a number they can double the number then add on the original number. Madeline has talked with many of the parents to explain this new approach and says that once she explains the thinking behind a strategies approach to the multiplication facts and the benefits for student learning the parents are very supportive. This focus on strategies in the learning of multiplication facts is consistent with the way that Madeline now encourages students to explain their thinking in maths more generally. For example, her class do a short subitising activity each day and then explain how they partitioned the numbers to find the total. Depending on the way that the objects are arranged students might see 8 objects as 'a three and a five' or 'two fours' or 'two threes and a two'. Students share their strategies and soon appreciate that numbers are not fixed, they can be broken apart and put back together again and that there are lots of ways to do this. This understanding flows into strategies for mental addition so that for example, 18 + 27 could be 18 + 30 – 3 or 18 + 2 + 25 depending on which mental image is more powerful for the individual student.

Madeline uses the Big Ideas in Number diagnostic tools regularly to identify gaps in student learning. Previously, she says, some children were able to 'hide' or bluff their way along – they appeared to be learning or at least they didn't stand out as not learning. When using one of the diagnostic tools however students cannot bluff their way through, she knows what they do or do not understand. Madeline finds the Advice section particularly helpful to ensure that she focuses new learning appropriately for each student.

Madeline has reflected back on her introduction to the Big Ideas and says that she initially found the new language, terms like subitising, renaming, trusting the count, quite daunting but with Brianna's help she persevered. She now finds it easier to have discussions with colleagues because if they talk about 'renaming' for example they all know what is meant. The students too have enjoyed playing around with new language. For example, when working on place value the students really enjoyed playing with the language and using made up names like 'onety-one, onety-two' and so on.

As an early career teacher Madeline has found working on the Big Ideas in Number with the support of a Numeracy Coach to be a huge boost to her confidence as a teacher of mathematics. In her first year of teaching in 2009 she worried that her lack of confidence with maths would spill over into the learning of her students. Now she says,

> I'm so glad I have worked with the Big Ideas in Number as a young teacher and that it is now becoming second nature. When I started teaching I moved on too quickly without really building strong foundations of understanding. I can now look at the child not just the curriculum. I can target learning to my students' needs not just teach Year 3 or Year 4. I started off dreading maths but now I enjoy teaching maths more than anything.

Guy is an experienced teacher in a metropolitan primary school with a Year 5/6 class. Guy and Chris, the Numeracy Coach, have been working together on the Big Ideas since late 2009 and over this period they have used the diagnostic tools and advice and introduced the students to a range of learning games and activities designed to build on students' immediate learning needs.

Over time they have developed many other activities that have spring-boarded from the original advice. One of these is using the additive strategies cards (see Figure 1 below) that are projected on the smart-board.

33	18
22	?

?	12
12	48

Figure 1. Additive strategies cards (adapted from Siemon, 2006)

Students record their answers and the way they worked it out. They then share their strategies (e.g., doubling, near doubles, make to the nearest ten, number splitting and compensating) and discuss the relative efficiency of each. Teachers and students invent new and interesting variations of these 4 square problems and as students become more efficient the problems become more difficult!

Since working with the big ideas Guy has placed more emphasis on children talking about their maths learning and recording their thinking in as many ways as possible. This approach helps develop mathematical language but also as students learn about successful approaches from their peers they increase their flexibility in working with number. He has implemented a range of classroom strategies and protocols to support this emphasis. One example is the use of a laminated A3 sheet on which the students record their thinking in words, symbols or diagrams. As this work is easily changed or erased students are more inclined to write and record than they are on paper or in a maths book. The students can hold up their laminated sheets to help explain their thinking to other students and discuss the accuracy and efficiency of their strategies.

Chris and Guy have used some of the Big Ideas in Number diagnostic tools as whole class or group activities. For example, an activity they use is called Thinking Strings where a student or teacher randomly places a line of magnetic base 10 blocks on the white board as shown. Students then count on in place value parts and record their thinking as shown in Figure 2.

Figure 2. Use of thinking strings and base 10 blocks

When students are involved in these activities Chris and Guy have opportunities to sit with individual students, observe their responses, probe their thinking and give immediate constructive feedback.

In working with the language of the *Big Ideas in Number*, Guy and Chris have become aware that for many students talking about mathematics strategies and learning is difficult. These students need explicit scaffolding and support to develop their mathematical vocabulary and the confidence to use it in front of their classmates.

The Tasmanian Experience

Over the past two years interest in improving student outcomes in mathematics has grown in many Tasmanian schools. Professional learning focusing on the big ideas in number (Siemon, 2006) was seen by the Tasmanian Department of Education as a means of focusing teachers' attention on what is important and to encourage school communities to recognise the importance of developing strong foundations in number for all strands of the mathematics curriculum.

Teachers and school mathematics leaders from all parts of the state have participated in professional development workshops and there has been a very high level of take up of the *Assessment for Common Misunderstandings* materials sourced and used with permission from DEECD.

Many teachers have been surprised about the misunderstandings their students have and they have realised the gaps in student understanding which have contributed to poor performance in NAPLAN and other assessments. For example, Maree who teaches a grade 5/6 class realised that some of her students could not subitise small collections (ACMNA003) and solved all numerical problems by counting by ones. This gave her valuable information about where to target her teaching with this group of students. Using a flip camera she was able to capture some students while they were assessed using the Trust the Count assessment tool and the video footage has been used in several professional learning sessions to help other teachers unpack the assessment tools and to realise that they too may have students who struggle with ideas well below where they might expect them to be in an upper primary class.

Amanda who teaches grade 5/6 found that she had students who struggled with early ideas in place value. She used the place value tools to probe student understanding and the advice from the website to differentiate instruction to address the student's learning needs. Teaching groups were formed to target intervention and pre and post testing showed significant gains in student understanding.

Similarly, school mathematics leaders Pam and Sylvia from a large primary school note that:

> The big ideas provide our teachers with a focus on the important ideas in mathematics. The big ideas leave no doubt about what is sequentially important for students to know and understand. They stress the importance of building mental images and visualising concepts. The focus is on hands-on learning that promotes deep understanding of maths concepts and on building the language of maths. The Assessment for Common Misunderstandings tools are being developed into kits and will support teachers in assessing students but will also provide teachers with a "where to next?' for planning for learning. The big ideas will underpin our whole school approach document in conjunction with the Australian Curriculum.

With the decision to fully implement the ACM in 2012 in all Tasmanian Schools it became obvious that teachers need good pedagogical content knowledge and access to assessment tools which support their curriculum decision-making in response to student learning needs.

> Curriculum-related assessment information is required for a detailed analysis of students' learning needs. These kinds of data are more useful for the purposes of diagnosing students' learning needs than assessments focused more on identifying normative achievement, but not related to the curriculum (Timperley, 2009, p. 22)

As a consequence, in 2011, 11 schools in one geographic region of the state used the big ideas in number and data derived from the associated tools to focus teacher professional learning on meeting student learning needs. The best evidence synthesis of effective professional learning for teachers (Timperley, 2007) was used as a foundation for the project. Teachers adopted an inquiry approach to their teaching and professional learning based on identified student needs and data, not generic professional learning based on what the Tasmanian Department of Education might *think* teachers need. This approach was framed by an adaptation of Timperley's (2009) teacher inquiry and knowledge-building cycle (see Figure 3) on the grounds that while high quality assessment is valuable, "much more is needed to improve teaching practice in ways that have a substantive impact on student learning" (p. 21).

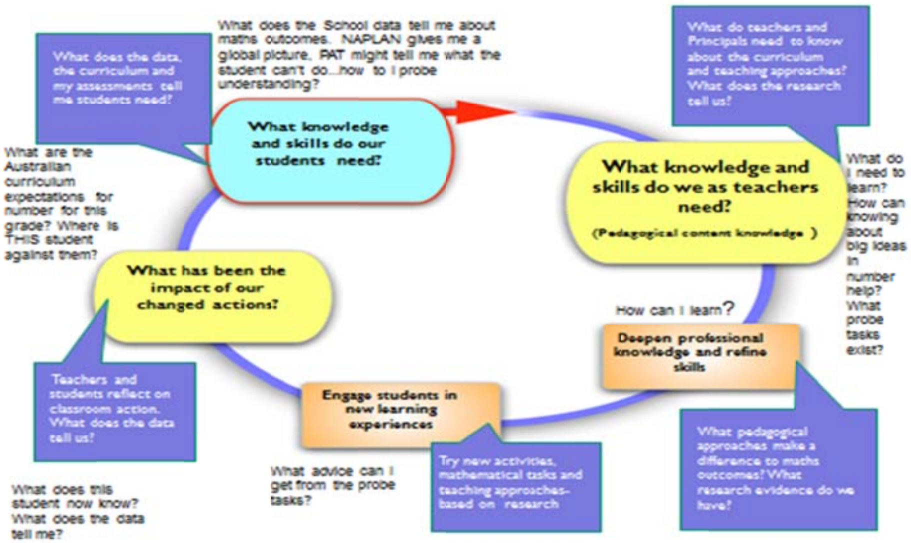

Figure 3. Teacher inquiry and knowledge building cycle (adapted from Timperley, 2009)

Project schools were provided with a full set of the tools and teachers involved in the project worked with school-based lead teachers and external facilitators to engage in data-driven conversations to determine professional learning needs and teaching focus. The big ideas framework and associated assessment tools enabled teachers to go beyond the data they may have gathered from other sources (e.g., NAPLAN) to delve into student misconceptions in number and use the advice to teach in richer, more focussed and intentional ways (e.g., see Figure 4).

Figure 4. Children working with ten-frames and bead strings.

This work will be invaluable in building teacher capacity to focus assessment and teach for the depth and understanding which is a key pedagogical underpinning of the mathematics curriculum, for example, "it is preferable for students to study fewer aspects in more depth rather than studying more aspects superficially" (National Curriculum Board, 2009, p. 14). This will also help teachers become more aware of the finer detail of the key ideas underpinning the content descriptors in the ACM which are often very broad and open to interpretation. As stated in the *Shape of the Australian Curriculum: Mathematics* (National Curriculum Board, 2009), "teachers can make informed classroom decisions interactively if they are aware of the development of key ideas" (p. 14).

John, who is a facilitator/coach in the project and a former secondary mathematics teacher, confirms this view when he says:

> the big ideas have focused my thinking around the sequential development of ideas. To address the diverse ability range of our students, teachers need to understand how mathematical concepts are developed and teachers need to be supported to translate this into improved classroom practice.

His colleague Wendy, with a background in Early Childhood education concurs when she reflects on how her work has altered since she has focused on big ideas in number:

> Now that I have a deeper understanding of [the] big ideas it has made me more aware of the possible misunderstandings that children can have in key areas of number. A huge change in my thinking was realising that as an ECE classroom teacher that I hadn't been taking children back far enough when designing intervention programs. It was a struggle to get them to learn and apply strategies and when faced with solving problems they would always count on in ones.
>
> This has impacted greatly on my messages to ECE teachers now:
>
> - Spend more time developing mental images especially with subitising tasks (it is crucial...then hopefully we won't have children in Grade 6 and beyond still counting on in ones!)
> - Teachers of Grade 1 and Grade 2 should spend more time on and give children more opportunities for developing Place Value concepts...Counting collections and recording the number, bundling etc

As teachers involved in this project further explore the ACM and its focus on the four proficiencies, there is potential for re-visiting the big ideas in number in new and exciting ways, emphasising the explicit teaching focus for teachers and the importance of tasks that are selected to focus on both content descriptors and proficiencies.

Tasmanian schools have seen the potential of teacher professional learning based on in-depth knowledge of student understanding of key ideas in number. The big ideas in number provide a valuable framework for exploring questions such as "what is important?" "What will give us the greatest leverage in improving student outcomes?" Indeed, these big ideas are influencing our small state and engaging teachers in new learning for themselves and their students!

Conclusion

This chapter considered why big ideas have become a focus of attention in recent years in relation to the teaching and learning of mathematics and the design of school mathematics curriculum. It was suggested that this is a response to evidence from international assessments and large-scale numeracy research projects that many students in the middle years lack the depth of knowledge needed to critically apply mathematics. A focus on 'big ideas' and the links between them is needed to highlight key ideas and strategies at different levels of schooling, thin-out the overcrowded curriculum, and help deepen teacher knowledge and confidence to support more targeted teaching approaches. This is particularly the case for Number which has been shown to be the area most responsible for the range in mathematics achievement in the middle years.

Not everyone will agree with the notion of big ideas presented here, that is, as important organizing frames for thinking about and working with mathematics without which student progress in mathematics will be seriously impacted. This view motivated the choice of the big ideas in number used to inform the design of the *Assessment for Common Misunderstandings* materials, five of which are considered in the chapter, namely, trusting the count, place-value, multiplicative thinking, partitioning, and proportional reasoning. While these provided a useful lens to examine the ACM it would be naïve to think that a national mathematics curriculum could be organised in terms of these big ideas. Curricula serve many purposes and have many audiences but it is not unreasonable to suggest that in implementing the ACM and thinking about the type of professional learning needed to support a 21st century mathematics curriculum, serious consideration be given to these overarching themes and how these relate to and help connect the many seemingly disjointed behaviours that inevitably have to be listed in a document such as the ACM.

The South Australian and Tasmanian experiences of using the tools to inform and better target teaching practice indicate that a focus on the big ideas in Number 'works'. 'It's not rocket science' - teacher feedback on the use of the tools report significant improvements in student engagement and progress where student learning needs in relation to a small number of 'really big ideas' in Number are more accurately identified and the teaching is more closely targeted to meeting those needs.

Acknowledgement

The authors would like to acknowledge Claudia Johnstone who read earlier versions of this chapter and provided valuable feedback.

References

Askew, M. (1999). It ain't (just) what you do: Effective teachers of numeracy. In I. Thompson (Ed.), *Issues in teaching numeracy in primary schools*. Buckingham, UK: Open University Press.

Australian Association of Mathematics Teachers. (2009, May). *School Mathematics for the 21st Century. Discussion paper*. Adelaide: AAMT

Australian Curriculum, Assessment and Reporting Authority [ACARA]. (2011). Australian Curriculum: Mathematics. Retrieved from http://www.australiancurriculum.edu.au/Mathematics/Rationale

Australian Education Council. (1991). *A National statement on mathematics for Australian schools.* Carlton, Vic: Curriculum Corporation

Bond, T. & Fox, C. (2001). *Applying the Rasch model: Fundamental measurement in the human sciences*. Mahweh, NJ: Lawrence Erlbaum Associates

Callingham, R. (2011, July). Mathematics assessment: Everything old is new again? In J. Clark, B. Kissane, J. Mousley, T. Spencer & S. Thornton (Eds.), *Mathematics: Traditions and practices*, Proceedings of the 23rd Annual Conference of the Mathematics Education Research Group of Australasia, (pp. 134-141). Alice Springs: MERGA.

Charles, R. (2005). Big ideas and understandings as the foundation for elementary and middle school mathematics. *Journal of Education Leadership, 7*(3), 9-24.

Clarke, D. M., & Clarke, B. A. (2002). Challenging and effective teaching in junior primary mathematics. In M. Goos & T. Spencer (Eds.), *Mathematics: Making waves* Proceedings of the 19th Biennial Conference of the Australian Association of Mathematics (pp. 309-318). Adelaide: AAMT.

Clarke, F. & Kamii, C. (1996). Identification of multiplicative thinking in children in grades 1- 5. *Journal for Research in Mathematics Education, 27*(1), 41-51.

Clements, D. & Samara, J. (2007). Early childhood mathematics learning. In F. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 461-556). Charlotte, NC: Information Age Publishing.

Commonwealth of Australia (2008, May). *National Numeracy Review Report*. Commissioned by the Human Capital Working Group. Canberra, Commonwealth of Australia.

Confrey, J., Maloney, A., Nguyen, K., Mojica, G. & Myers, M. (2009). Equipartitioning/splitting as a foundation for rational number reasoning using learning trajectories. In M. Tzekaki, M. Kaldrimidrou & C. Sakondis (Eds.). *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education*, Vol. 2, pp. 345-352.

Hattie, J. (2003, October) *Teachers make a difference. What is the research evidence?* Paper presented to the annual research conference of the Australia Council for Education Research, Melbourne: ACER. Retrieved from http://www.acer.edu.au/documents/RC2003_Proceedings.pdf

Killion, K., Steffe, L. & Stanic, G. (1989). Children's multiplication. *Arithmetic Teacher*, 1(37), 34-36.

Kuntze, S., Lerman, S., Murphy, B., Kurz-Milcke, E., Siller, H. & Winbourne, P. et al. (2009). *Awareness of big ideas in mathematics classrooms – ABCmaths Progress report* (public part). Ludwigsburg University of Education, EACEA.

Lakoff, G. (1987). Cognitive models and prototype theory. In U. Neisser (Ed.) *Concepts and conceptual development* (pp. 63-100). Cambridge University Press: Cambridge

Lamon, S. (1999). *Teaching fractions and ratios for understanding – Essential content knowledge and instructional strategies for teachers*. Mahwah, NJ: Lawrence Erlbaum Associates.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States.* Mahwah, N.J.: Lawrence Erlbaum.

McIntosh, A. & Dole, S. (2004). *Mental computation: A strategies approach*. Hobart: Department of Education Tasmania.

Ministerial Council on Education, Employment, Training and Youth Affairs (2008). *The Melbourne Declaration on Educational Goals for Young Australians.* Melbourne: MCEETYA.

National Mathematics Advisory Panel. (2008). *Foundations for success: Final report of the National Mathematics Advisory Panel*. Washington: US Department of Education. Retrieved from http://www.ed.gov/MathPanel

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

National Curriculum Board. (2009). *Shape of the Australian Curriculum: Mathematics*. Melbourne: Commonwealth of Australia. Retrieved from http://www.acara.edu.au/publications.html

Nunes, T. & Bryant, P. (1996). *Children doing mathematics*. Oxford, UK: Blackwell

Ontario Ministry of Education. (2006). *Number sense and numeration, Grades 4 to 6*, Vols. 1-6. Toronto: Ontario Department of Education. Retrieved from http://www.eworkshop.on.ca/edu/resources/guides/NSN_vol_1_Big_Ideas.pdf

Ross, S. (1989). Parts, wholes, and place value: A developmental view. *Arithmetic Teacher* , 36, 47-51.

Siemon, D. (2006). *Assessment for Common Misunderstandings Materials*. Prepared for and published electronically by the Victorian Department of Education and Early Childhood Development. Retrieved from http://www.education.vic.gov.au/studentlearning/teachingresources/maths/common/default.htm

Siemon, D. (2011a). *Developmental maps for number P-10*. Materials commissioned by the Victorian Department of Education and Early Childhood Development, Melbourne.

Siemon, D. (2011b). Realising the big ideas in number – Vision impossible? *Curriculum Perspectives, 31*(1), 66-69.

Siemon, D., Beswick, K., Brady, K., Clark, J., Faragher, R., & Warren, E. (2011). *Teaching Mathematics: Foundations to the middle years.* Melbourne: Oxford University Press.

Siemon, D., Breed, M., Dole, S., Izard, J., & Virgona, J. (2006). *Scaffolding Numeracy in the Middle Years – Project Findings, Materials, and Resources*, Final Report submitted to Victorian Department of Education and Training and the Tasmanian Department of Education, Retrieved from http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/snmy.ppt

Siemon, D., Enilane, F. & McCarthy, J. (2004). Supporting Indigenous students' achievement in numeracy. *Australian Primary Mathematics Classroom*, *9*(4), 50-53.

Siemon, D., Virgona, J. & Corneille, K. (2001). *The Final Report of the Middle Years Numeracy Research Project: 5-9*. Retrieved from http://www.eduweb.vic.gov.au/edulibrary/public/curricman/middleyear/MYNumeracyResearchFullReport.pdf

Smith, J. & Thompson, P. (2007). Quantitative reasoning and the development of algebraic reasoning. In J. Kaput, D. Carraher, & M Blanton (Eds.), *Algebra in the early grades* (pp. 95-132). New York: Erlbaum.

Steffe, L. P., Cobb, P., & von Glasersfeld, E. (1988). *Young children's construction of arithmetical meanings and strategies*. New York: Springer-Verlag.

Steinle, V. & Stacey, K. (2004). A longitudinal study of students' understanding of decimal notation: An overview and refined results. In I. Putt, R. Farragher & M. MacLean (Eds.), *Mathematics Education for the third millennium: towards 2010. Proceedings of the 27th annual conference of the Mathematics Education Research Group of Australasia* (pp. 541-548).Townsville: MERGA.

Thomson, S., de Bortoli, L.,Nicholas, M., Hillman, K., Buckley, S. (2011) *Challenges for Australian Education: Results from PISA 2009*. Camberwell, VIC: Australian Council for Educational Research.

Timperly, H. (2007) *Teacher Professional Learning and Development: Best Evidence Synthesis Iteration (BES).* Retrieved from http://www.educationcounts.govt.nz/publications/series/2515/15341

Timperley, H. (2009, August). *Assessment and student learning: Collecting, interpreting and using data to inform teaching.* Presentation to the Annual Research Conference of the Australian Council for Educational Research, Perth. Retrieved from http://research.acer.edu.au/research_conference/RC2009/17august/20/

Vergnaud, G. (1983). *Multiplicative structures*. In R. Lesh & M. Landau (Eds.), Acquisition of mathematics concepts and processes (pp. 127-173). New York: Academic Press

Vincent, J. & Stacey, K. (2008). Do mathematics textbooks cultivate shallow teaching? Applying the TIMSS Video Study criteria to Australian eighth-grade mathematics textbooks. *Mathematics Education Research Journal, 20*(1), 81-106.

Willis, S. (2002). Crossing Borders: Learning to count. *Australian Educational Researcher, 29*(2), 115-130.

Authors

Dianne Siemon, RMIT University.
Email: dianne.siemon@rmit.edu.au

John Blekly, South Australian Department of Education and Childhood Services.

Denise Neal, Tasmanian Department of Education.